30 Years Working with Jaffre and Roberts on Modeling Flow in Porous Media

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Selected Jaffre/ Roberts Computational Research Contributions


- Mixed and Hybrid Methods, Roberts and Thomas, (1991)

- Upstream Weighting and Mixed Finite Elements in Simulation of Miscible Displacements, Jaffre and Roberts (1983)

- On Upstream Mobility Schemes for 2-Phase Flow in Porous Media, Mishra and Jaffre


- Modeling Fractures and Barriers as Interfaces for Flow in Porous Media, Martin, Jaffre, Roberts (2005)

- Godunov Type Methods for Conservation Laws with a Flux Function Discontinuous in Space
Societal Needs in Relation to Geological Systems

Resources Recovery
- Petroleum and natural gas recovery from conventional/unconventional reservoirs
- \textit{In situ} mining
- Hot dry rock/enhanced geothermal systems
- Potable water supply
- Mining hydrology

Waste Containment/Disposal
- Deep waste injection
- Nuclear waste disposal
- \( \text{CO}_2 \) sequestration
- Cryogenic storage/petroleum/gas

Underground Construction
- Civil infrastructure
- Underground space
- Secure structures

Site Restoration
- Aquifer remediation
- Acid-rock drainage
Mojdeh Delshad, Changli Yuan, Andro Mikelic, Ivan Yotov, Thomas Wick, Gergina Pencheva, Vivette Girault, Kundan Kumar, Gurpreet Singh.

Jaffre/ Roberts: Mixed Methods, Multiphase Flow, Reactive Transport, Miscible Displacement and Fingering, DG, Fracture Modeling,
Outline (Work Motivated by Jaffre/Roberts)

- Multipoint Flux Mixed Finite Element Method (MFMFE) for Flow and Coupling with Geomechanics
  - Example: poroelasticity with fixed fractures

- Chemical EOR: Polymer Flow and ASP (alkaline, surfactant, polymer)

- EOS Compositional Flow
  - Formulation
  - Brugge Co2 EOR
  - Coupling with EnKF for In Salah Co2 Sequestration

Conclusions
Single Phase Flow

\[ u = -K \nabla p \quad \text{in } \Omega, \]
\[ \nabla \cdot u = f \quad \text{in } \Omega, \]
\[ p = 0 \quad \text{on } \partial \Omega, \]

\[ (K^{-1}u_h, v)_Q - (p_h, \nabla \cdot v) = 0, \quad \forall v \in V_h \]
\[ (\nabla \cdot u_h, q) = (f, q), \quad \forall q \in W_h \]

• Q represents the quadrature rule
Corner Point Geometry - Highly Distorted Hexahedra
Multipoint Flux Mixed Finite Element

- Provably **accurate**:
- Pressure to second order;
- Velocity to first order.
- Locally **mass conservative**.
- Easy to implement.
- Current Extensions:
  - Non-isothermal compositional model.
  - Nonplanar fractured grids.
Fractured Reservoir Flow Model

- Interface as pressure specified BC for reservoir
- No-flow BC for fracture
- Jump in reservoir flux across interface as the source term for fracture
Model Formulation

Reservoir Flow
\[ \frac{\partial}{\partial t} \left( \phi^* S_\beta \rho_\beta \right) + \nabla \cdot z_\beta = q_\beta \]
\[ z_\beta = -K \rho_\beta \frac{k_{r\beta}}{\nu_\beta} (\nabla p_\beta - \rho_\beta g) \]

Fracture Flow
\[ \frac{\partial}{\partial t} \left( w S_\beta^\Gamma \rho_\beta^\Gamma \right) + w \nabla \cdot z_\beta^\Gamma = q_\beta^\Gamma + q_\beta \]
\[ z_\beta^\Gamma = -K^\Gamma \rho_\beta^\Gamma \frac{k_{r\beta}}{\nu_\beta} (\nabla p_\beta^\Gamma - \rho_\beta^\Gamma g) \]

Interface Conditions
\[ z_\beta \cdot n = 0 \text{ on } \partial \Omega^N \]
\[ p_{ref} = p^D \text{ on } \partial \Omega^D \]
\[ S_{ref} = S^D \text{ on } \partial \Omega^D \]
\[ p_{ref} = p^D \text{ on } \Gamma^\pm \]
\[ S_{ref} = S^D \text{ on } \Gamma^\pm \]
\[ q_\beta = [z_\beta \cdot n]^\Gamma = z_\beta \cdot n|_{\Gamma^-} - z_\beta \cdot n|_{\Gamma^+} \]
\[ w = [u \cdot n]^\Gamma = u \cdot n|_{\Gamma^-} - u \cdot n|_{\Gamma^+} \]
\[ K^\Gamma = \frac{w^2}{12} \]
• Coupling of standard Biot of linear poroelasticity and flow (iterative coupling—Mikelic, W) in fracture governed by lubrication (Kumar, W)

• Theorem: Existence and uniqueness and a priori results established for coupled linearized system under weak assumptions on data. Error estimates also derived. (Girault, W, Ganis, Mear)
A Lubrication Fracture Model in a Poro-Elastic Medium

- Darcy’s Law (reservoir flow), Linear Elasticity (reservoir mechanics), and Reynold’s Lubrication (fracture flow).
- Multipoint flux mixed finite elements on hexahedra.
- Solution algorithm uses iterative coupling.

- Unknowns include width, leakoff, traction.
- Existence and uniqueness were proven.
- Has been extended to multiphase flow in IPARS.
Motivation for Chemical EOR Studies

- Improve oil recovery efficiency for displacements with unfavorable mobility ratio and very heterogeneous reservoirs
- Target bypassed oil left after waterflood
- Reduce mobility ratio to improve areal and vertical sweep efficiencies
- Compare efficiency/accuracy of different numerical schemes (IMPES, IMPLICIT, Iterative Coupling, Time splitting)
- Process scale up to field scale
- Chemical EOR in fractured porous media, e.g., Alaska
Improved Mobility & Sweep Efficiency

Figure 8.2. Schematic diagram of the improvement of areal sweep caused by polymer flooding in a five-spot system.
Polymer Structure

Large chains of repeating monomers linked by covalent bonds

Xanthan (MW ~ 2-50 MM)

Polyacrylamide (MW ~ 2-30 MM)

ANIONIC AND NON-IONIC POLYACRYLAMIDES

Copolymers of acrylamide and acrylic acid

\[
\begin{align*}
\text{Acrylamide} & \quad \text{Acrylic acid} \\
m \ CH_2=CH + n \ CH_2=CH + \text{NaOH} \rightarrow \quad \left( CH_2\text{CH} \right)_m \\
\quad \quad \quad \text{C} &= \text{O} \\
\text{C} &= \text{O} \\
\text{NH}_2 & \quad \text{OH} \\
\end{align*}
\]

Remark: with \( m = 0 \), the polyacrylamide is non-ionic
Mobility Ratio

The ratio of displacing fluid mobility to displaced fluid mobility:

\[ M = \frac{\lambda_W}{\lambda_O} = \frac{k_W}{k_O} \frac{\mu_W}{\mu_O} = \frac{k_W \mu_O}{k_O \mu_W} \]

\[ M \leq 1 \quad \text{Piston-like displacement} \]

Small amount of polymer increases water viscosity

Source: Lake, 1989
Polymer Rheology

Dilute polymer solutions are pseudoplastic (shear thinning)

Figure 3.5. Different types of shear stress/shear rate behaviour found in polymeric fluids; the elastic solid and ideal fluid cases are also shown.
<table>
<thead>
<tr>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two phase oil/water</td>
</tr>
<tr>
<td>Compressible fluids</td>
</tr>
<tr>
<td>MFMFE Based</td>
</tr>
<tr>
<td>Time split method for flow and concentration (transport, diffusion/dispersion)</td>
</tr>
<tr>
<td>Non-differentiable inequality constraints – model as minimization of Gibbs free energy using interior pt.</td>
</tr>
<tr>
<td>Several boundary condition options</td>
</tr>
<tr>
<td>Wells as volumetric or pressure constraint</td>
</tr>
<tr>
<td>AMG solver with pre-conditioner</td>
</tr>
<tr>
<td>Parallel computation capability</td>
</tr>
<tr>
<td>General geochemistry and biochemistry modules</td>
</tr>
</tbody>
</table>
Polymer Properties in IPARS-TRCHEM

- Viscosity as a function of Concentration, Salinity, Shear rate
- Adsorption
- Permeability reduction
- Inaccessible pore volume
Chemical Flooding Modules

• Surfactant
  – Reduce the interfacial tension between oil and water phases
  – Target bypassed oil left after waterflood by mobilizing oil trapped in pores due to capillary pressure/force

• Polymer
  – Reduce water mobility to improve areal and vertical sweep efficiencies
  – Target bypassed oil left after waterflood due to unfavorable mobility ratio and heterogeneity

Model field-scale tests using parallel computation
Multiphase Flow Equations

- Mass Conservation for each phase
  \[ \frac{\partial (\phi \rho_\alpha S_\alpha)}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = q_\alpha \]

- Darcy’s Law: \[ \mathbf{u}_\alpha = -\frac{k_\alpha}{\mu_\alpha} \mathbf{K} (\nabla p_\alpha - \rho_\alpha g \nabla z) \]

- Saturation constraint: \[ \sum S_\alpha = 1 \]

- Capillary pressure: \[ P_c(S_w) = P_n - P_w \]
Reactive Species Transport Model

- Mass balance of species \( i \) in phase \( \alpha \):
  \[
  \frac{\partial (\phi c_{i\alpha} S_{\alpha})}{\partial t} + \nabla \cdot (c_{i\alpha} \vec{u}_\alpha - \phi S_{\alpha} \vec{D}_{i\alpha} \nabla c_{i\alpha}) = \phi S_{\alpha} R_{i\alpha}^C + q_{i\alpha}
  \]

- An equilibrium linear partition between phases
  \[c_{i\alpha} = \Gamma_{i\alpha} c_{ir}\]

- Phase-summed species transport equation:
  \[
  \frac{\partial (\phi^*_i c_{iw})}{\partial t} + \nabla \cdot (c_{iw} \vec{u}^*_i - \vec{D}^*_i \nabla c_{iw}) = q_i^T + R_{i}^{TC}
  \]
  \[
  \phi^*_i = \phi(S_w + \Gamma_{io} S_o) \quad \vec{u}^*_i = \vec{u}_w + \Gamma_{io} \vec{u}_o \quad q_i^T = q_w + \Gamma_{io} q_o
  \]
  \[
  \vec{D}^*_i = \phi(S_w \vec{D}_{iw} + S_o \Gamma_{io} \vec{D}_{io}) \quad R_{i}^{TC} = \phi(S_w R_{iw}^C + S_o R_{io}^C)
  \]
Component Transport Equations

- Mass balance of species $i$ in phase $\alpha$:

$$
\frac{\partial (\phi c_{i\alpha} S_{\alpha})}{\partial t} + \nabla \cdot (c_{i\alpha} u_{\alpha}) - \phi S_{\alpha} D_{i\alpha} \nabla c_{i\alpha} = q_{i\alpha}
$$

- The diffusion-dispersion tensor $D_{i\alpha}$ is given by:

$$
D_{i\alpha} = D_{i\alpha}^{\text{mol}} + D_{i\alpha}^{\text{hyd}}
$$

$$
D_{i\alpha}^{\text{hyd}} = f(\text{velocity})
$$
Validation against an IMPES Code

Total Inj. Rate Vs. Time : (inj)

Polymer Injection Concentration Vs. Time : (inj)

Bottom Hole Pressure Vs. Time : (inj)

Cum. Oil Rec. Vs. Time

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>q_{inj} (ft^3/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>600</td>
<td>1000</td>
</tr>
<tr>
<td>900</td>
<td>1500</td>
</tr>
<tr>
<td>1200</td>
<td>2000</td>
</tr>
<tr>
<td>1500</td>
<td>2500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>P_{BH} (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000</td>
</tr>
<tr>
<td>300</td>
<td>4000</td>
</tr>
<tr>
<td>600</td>
<td>6000</td>
</tr>
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</tr>
<tr>
<td>1200</td>
<td>10000</td>
</tr>
<tr>
<td>1500</td>
<td>12000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>Npd (fraction of OOIP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0.02</td>
</tr>
<tr>
<td>400</td>
<td>0.04</td>
</tr>
<tr>
<td>600</td>
<td>0.06</td>
</tr>
<tr>
<td>800</td>
<td>0.08</td>
</tr>
<tr>
<td>1000</td>
<td>0.1</td>
</tr>
<tr>
<td>1200</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>Cp (wt%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>300</td>
<td>0.02</td>
</tr>
<tr>
<td>600</td>
<td>0.04</td>
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<td>0.06</td>
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</tr>
<tr>
<td>1500</td>
<td>0.1</td>
</tr>
</tbody>
</table>

UTCHEM IPARS
Parallel Simulation of Polymer Injection

- 200 cp oil viscosity (endpoint mobility ratio = 107)
- Domain size: 10240 ft x 5120 ft x 160 ft
- Grid size: 20 ft x 10 ft x 10 ft
- No. of gridblocks: 4,194,304
- Average perm: (about 10 D)
- 32 five spots with 37.6 acre well patterns
- 32 injection wells and 45 production wells
- Constant pressure injection (below parting pressure)
- 128 processors
Polymer Flood Simulations

Permeability, md

Polymer conc.

Oil saturation
Polymerflood Recovery for Viscous Oil

Cumulative Oil Recovery (%OOIP)

- **WF 10cp**
  - 2000 days: 37.50 PV
  - 4000 days: 58.27 PV
- **PF 10cp**
  - 2000 days: 3.68 PV
  - 4000 days: 0.78 PV
- **WF 200cp**
  - 2000 days: 21 PV
  - 4000 days: 40.63 PV

**Note:** The diagram compares the recovery for different polymerflood scenarios at two time points, 2000 and 4000 days.
Parallel Scalability

CPU Time

![CPU time graph](image)

Parallel Scalability

![Parallel Scalability graph](image)
**ASP Model Species**

**Polymer flood**: 3+ species, the first 3 species must be polymer, anion (Cl⁻), cation (Ca²⁺)

**SP flood**: 4+ species, the first 4 species must be polymer, anion (Cl⁻), cation (Ca²⁺), surfactant

**ASP flood**: 12+ species, the first 12 species must be polymer, anion (Cl⁻), cation (Ca²⁺), surfactant, H⁺, HA₀, CO₃²⁻, Na⁺, Mg²⁺, A⁻, HAₕ, OH⁻
Alkaline/Surfactant/Polymer (ASP) Flood Flowchart

Initialize

NSTEP = NSTEP + 1

Solve two-phase flow

CSTEP = CSTEP + 1

Reactive transport: advection, diffusion & dispersion, geochemistry

Adsorption of polymer, surfactant, and alkaline

$t_{m+1} = t_{m} + \Delta t_{m}$

$t_{n+1} = t_{n} + \Delta t_{n}$

IFT, ME viscosity, trapping number, relative permeabilities

Soap generation, soap and surfactant phase behavior

Polymer properties: viscosity, permeability reduction

$t_{m} = t_{n} \,$?
Alkaline/Surfactant/Polymer Module Features

- Polymer, surfactant, and alkaline adsorptions
- Non-Newtonian polymer solution and micoreemulsion (ME) viscosities
- Permeability reduction and pore volume reduction
- In situ generation of soap by reaction of alkaline with the acid in crude oil
- Phase behavior as a function of soap and surfactant concentrations
- Aqueous geochemical reactions, mineral dissolution/precipitation, and ion exchange with clays in the rock and micelles
Field-scale unstable polymer flood

- Reservoir dimensions: 1024 x 256 x 256 (ft)
- Gridblocks in each direction: 128 x 64 x 128
- Gridblock sizes: 8 x 4 x 2 (ft)
- Total gridblocks: 1,048,576
- Number of processors: 64
- Simulation time: 100 Day
Field-scale unstable polymer flood (Cont.)

- Average permeability: 2100md
- Porosity: 0.23
- Oil viscosity: 2000cp
- 1 horizontal injector at the bottom with $P_{BH} = 15000\text{psi}$
- 1 horizontal producer at the top with $P_{BH} = 3000\text{psi}$
- Injection rate: about 2600~3000STB/Day
- Injected polymer conc.: 0.07497lbmol/ft$^3$ (0.12wt%)
Polymer Viscosity

Polymer Concentration: 0.12wt%
Salinity: 0.017meq/ml
Relative Permeabilities

The graph shows the relationship between water saturation ($S_w$) and relative permeabilities ($K_{rw}$ and $K_{ro}$). As $S_w$ increases from 0 to 1, $K_{rw}$ decreases while $K_{ro}$ increases.
Relative Permeabilities

![Graph showing relative permeabilities](image)

- $K_{rw}$
- $K_{ro}$

Sw
Permeability Distribution and Well Locations

Injector
Producer

Perm (md)
Simulation Results at 100 Day

[3D diagram of simulation results with labels for PWAT (psi) and SWAT values]
Simulation Results at 100 Day (Cont.)
Compositional Equations

Component Conservation Equation

\[
\frac{\partial}{\partial t} \left( \sum_{\alpha} \phi S_{\alpha} \rho_{\alpha} \xi_{i\alpha} \right) + \nabla \cdot \sum_{\alpha} \left( \rho_{\alpha} \xi_{i\alpha} u_{\alpha} - \phi S_{\alpha} D_{i\alpha} \cdot \nabla (\rho_{\alpha} \xi_{i\alpha}) \right) = \sum_{\alpha} q_{i\alpha}
\]

Darcy Phase Flux

\[
u_{\alpha} = -K \frac{k_{r\alpha}}{\mu_{\alpha}} (\nabla p_{\alpha} - \rho g)
\]

Define Component Flux

\[
F_{i} = -K \left( \sum_{\alpha} \rho_{\alpha} \xi_{i\alpha} \frac{k_{r\alpha}}{\mu_{\alpha}} (\nabla p_{\text{ref}} - \rho g) + \sum_{\alpha \neq \text{ref}} \rho_{\alpha} \xi_{i\alpha} \frac{k_{r\alpha}}{\mu_{\alpha}} \nabla p_{ca} \right)
\]

Modified Compositional Equations

\[
\frac{\partial}{\partial t} (\phi N_{i}) + \nabla \cdot F_{i} - \nabla \cdot \left( \sum_{\alpha} \phi S_{\alpha} D_{i\alpha} (\nabla \rho_{\alpha} \xi_{i\alpha}) \right) = q_{i}
\]
Closure & Constraints

Capillary Pressure

\[ p_{c\alpha} = p_\alpha - P_{\text{ref}} \]

Phase Behavior

\[ \rho_\alpha = \frac{p_\alpha}{Z_\alpha RT} \]
\[ \rho_w = \rho_{w_0} \exp \left[ C_w (P_{\text{ref}} + P_{cw} - P_{\text{ref, std}}) \right] \]

Rock Compressibility

\[ \phi = \phi_0 \left[ 1 + C_r (P_{\text{ref}} - P_{\text{ref, std}}) \right] \]

Saturation Constraint

\[ \sum_\alpha S_\alpha = 1 \]
\[ S_w = \frac{N_w}{\rho_w} \]
\[ S_o = \frac{(1 - \nu)}{\rho_o} \sum_{i=2}^{N_c} N_i \]
\[ S_g = \frac{\nu}{\rho_g} \sum_{i=2}^{N_c} N_i \]
Hydrocarbon Phase Behavior

Peng-Robinson Cubic EOS

\[ \bar{Z}_\alpha^3 - (1 - B_\alpha) \bar{Z}_\alpha^2 + (A_\alpha - 3B_\alpha^2 - 2B_\alpha) \bar{Z}_\alpha - (A_\alpha B_\alpha - B_\alpha^2 - B_\alpha^3) = 0 \]

\[ Z_\alpha = \bar{Z}_\alpha - C_\alpha \]

Rachford-Rice for phase mole fraction (\( \nu \))

\[ f = \sum_{i=2}^{N_c} \frac{(K_i^{\text{par}} - 1)z_i}{1 + (K_i^{\text{par}} - 1)\nu} = 0 \]

Iso-fugacity criteria for \( K_i^{\text{par}} \)

\[ g = \ln(\Phi_{i0}) - \ln(\Phi_{ig}) - \ln K_i^{\text{par}} = 0 \]

Gibbs energy minimization for phase stability

\[ dG_{\alpha,T,P} = \sum_{i=2}^{N_c} \left. \frac{\partial G}{\partial n_i} \right|_{\alpha,T,P} \ dn_i = h(Z_\alpha) \]
Component Flux

\[
\left\langle \frac{1}{\Lambda_i^k} K^{-1} F_i^{k+1} , v_h \right\rangle_{Q,E} - \left( P_{\text{ref},i}^{k+1}, \nabla \cdot v_h \right)_E = - \int_{\partial E \cap \partial \Omega} P_{\text{ref}} v_h \cdot n - \left( \frac{1}{\Lambda_i^k} \sum_{\alpha \neq \text{ref}} \rho_{\alpha,i}^{k+1} \xi_{\alpha,i}^{k+1} \lambda_{\alpha,i}^{k+1} \nabla P_{\alpha,i}^{k+1}, v_h \right)_E
\]

Component Conservation Equation

\[
\left( \phi_h^{k+1} N_i^{k+1}, w_h \right)_E + \left( \nabla \cdot F_i^{k+1}, w_h \right)_E - \left( \nabla \cdot \sum_{\alpha} \left\{ \phi_h^{k+1} S_{\alpha,i}^{k+1} D_{\alpha,i}^{k+1} \cdot \nabla \left( \rho_{\alpha,i}^{k+1} \xi_{\alpha,i}^{k+1} \right) \right\}, w_h \right)_E
\]

\[
= \left( q_i, w_h \right)_E + \left( \phi^{n} N_i^{n}, w_h \right)_E.
\]

- Enhanced BDDF\textsubscript{1} mixed finite element space
- Symmetric and non-symmetric quadrature rules (Q)
- 9 and 27 point stencil for 2 and 3 dimensions, respectively
- \( \Lambda_i^k \)s are positive quantities
Diffusion-Dispersion

Full Tensor Diffusion-Dispersion

\[ D_{i\alpha} = D_{i\alpha}^{\text{mol}} + D_{i\alpha}^{\text{hyd}} \]
\[ D_{i\alpha}^{\text{mol}} = \tau_{\alpha} d_{m,i\alpha} I \]
\[ D_{i\alpha}^{\text{hyd}} = d_{t,\alpha} |v_{\alpha}| I + (d_{l,\alpha} - d_{t,\alpha}) v_{\alpha} v_{\alpha}^T / |v_{\alpha}| \]

Diffusive-Dispersive Flux Calculation

\[ J_{i\alpha} = \phi S_{\alpha} D_{i\alpha} \rho_{\alpha} \cdot \nabla (\xi_{i\alpha}) \]
\[ \left< \frac{1}{\phi \rho_{\alpha} S_{\alpha}} D_{i\alpha}^{-1} J_{i\alpha}, v_h \right>_{Q,E} - (\xi_{i\alpha}, \nabla \cdot v_h)_E = - \int_{\partial E \cap \partial \Omega} \xi_{i\alpha} v_h \cdot n. \]

- Accurate dispersion tensor calculation using flux vector at each corner
- Reduced grid-orientation effect on concentrations
Linearized Form

Component Flux

\[ \left\langle \frac{1}{\Lambda_{i,h}} K^{-1} \delta F_{i,h}, v_h \right\rangle_{Q,E} - (\delta P_{\text{ref},h}, \nabla \cdot v_h)_E = -R_{3i} \]

Component Mass Conservation

\[ \left( \frac{\phi_{h,n+1,k}^i \delta N_{i,h}}{\Delta t}, w_h \right)_E + \left( \frac{N_{i,h}^{n+1,k}}{\Delta t} \frac{\partial \phi}{\partial P_{\text{ref},h}} \delta P_{\text{ref},h}, w_h \right)_E + (\nabla \cdot \delta F_{i,h}, w_h)_E = -R_{4i} \]

\[
\begin{pmatrix}
A_i & B & 0 \\
B^T & C_i & D_i
\end{pmatrix}
\begin{pmatrix}
\delta F_i \\
\delta P_{\text{ref}} \\
\delta N_i
\end{pmatrix}
= 
\begin{pmatrix}
-R_{3i} \\
-R_{4i}
\end{pmatrix}
\]

- Eliminate fluxes \( \delta F_i \) to obtain a linear system of equations in \( \delta P \) and \( \delta N_i \)
Linearized Form

Saturation Constraint

\[ \sum_{\alpha} \frac{\partial S_{\alpha}}{\partial p_{\text{ref}}} \delta p_{\text{ref}} + \sum_{\alpha} \sum_{i} \frac{\partial S_{\alpha}}{\partial N_{i}} \delta N_{i} + \sum_{\alpha} \sum_{i} \frac{\partial S_{\alpha}}{\partial \ln K_{i}^\text{par}} \delta \ln K_{i}^\text{par} + \sum_{\alpha} \frac{\partial S_{\alpha}}{\partial \nu} \delta \nu = 1 - \sum_{\alpha} S_{\alpha} = -R_{5} \]

Fugacities at Equilibrium

\[ \frac{\partial \ln \Phi_{\text{io}}}{\partial p_{\text{ref}}} \delta p_{\text{ref}} + \sum_{k=2}^{N_{c}} \frac{\partial \ln \Phi_{\text{io}}}{\partial N_{k}} \delta N_{k} + \sum_{k=2}^{N_{c}} \frac{\partial \ln \Phi_{\text{io}}}{\partial \ln K_{k}^\text{par}} \delta \ln K_{k}^\text{par} + \frac{\partial \ln \Phi_{\text{io}}}{\partial \nu} \delta \nu - \left( \frac{\partial \ln \Phi_{\text{ig}}}{\partial p_{\text{ref}}} \delta p_{\text{ref}} \right) \]

\[ + \sum_{k=2}^{N_{c}} \frac{\partial \ln \Phi_{\text{ig}}}{\partial N_{k}} \delta N_{k} + \sum_{k=2}^{N_{c}} \frac{\partial \ln \Phi_{\text{ig}}}{\partial \ln K_{k}^\text{par}} \delta \ln K_{k}^\text{par} + \frac{\partial \ln \Phi_{\text{ig}}}{\partial \nu} \delta \nu \right) - \frac{\partial \ln K_{i}^\text{par}}{\partial \ln K_{k}^\text{par}} \delta \ln K_{k}^\text{par} = -R_{6i} \]

\[ \begin{pmatrix} E & F & G & H \\ I & J & K & L \\ 0 & N & O & P \end{pmatrix} \begin{pmatrix} \delta p_{\text{ref}} \\ \delta N \\ \delta \ln K_{i}^\text{par} \\ \delta \nu \end{pmatrix} = \begin{pmatrix} -R_{5} \\ -R_{6} \\ -R_{7} \end{pmatrix} \]

- Eliminate fluxes \( \delta K_{i}^\text{par} \) and \( \delta \nu \) to obtain a linear system of equations in \( \delta P \) and \( \delta N_{i} \)
Brugge Field Study

Brugge field geometry and well locations
Reservoir Properties

- 9x48x139 general hexahedral elements
- In-situ hydrocarbon fluid composition: 40% C$_6$, 60% C$_{20}$
- Injected fluid composition: 100% CO$_2$
- Initial reservoir pressure: 1500 psi
- 30 bottom-hole pressure specified wells
  - 10 injectors at 3000 psi
  - 20 producers at 1000 psi
- Initial water saturation: $S_w = 0.2$
- $\phi \approx 0.14 - 0.24$, $K_z = K_y$, $T_{res} = 160$ F
Rock Properties

Relative Permeability Curves

Capillary Pressure Curves
Pressure & Concentration Profiles

Pressure and concentration profiles after 1000 days
Saturation Profiles

- Multi-contact miscible flood
- Miscibility achieved at the tail end of the displacement front
Hydraulic Fracturing Stages

- Fracture growth: slick water injection
  - Length
- Proppant placement: polymer injection
  - Width due to polymer injection
  - Thickness due to proppant

Compaction
Proppant Placement

- Proppant
- Polymer
- Slick Water
Well Model Updates

- Multistage hydraulic fractures in a single well bore
Characteristics

- Polymer front travels ahead of proppant front
- Initial fracture thickness due to fracture growth during slick water injection
- Intermediate thickness increase due to fluid pressure front ahead of proppant front
- Final thickness related to proppant concentration
- Compaction related width changes
Phase Field for Crack Propagation (Mikelic, W, Wick)

Four advantages

- Fixed-mesh approach avoiding remeshing
- Crack nucleation, propagation and path are included in the model avoiding evaluation of stress intensity factors
- Joining and branching of multiple cracks easy to realize
- Cracks in heterogeneous media
Energized Fractures
In Salah Reservoir

- Salah Gas Project in Algeria is world’s first industrial scale CO2 storage project in depleting gas field
- Aprox. 0.5-1 M tons CO2 per year injected since August 2004
- Aquifer: low-permeability, 20 m thick carboniferous sandstone, 1800-1900 m deep
In Salah Reservoir

- Three long-reach (about 1-1.5 km) horizontal injection wells
- Satellite-based interferrometry (InSAR) has been used for detecting ground surface deformations related to the CO2 injection
- Uplift occurred within a month after start of the injection and the rate of uplift was approximately 5 mm per year (~2 cm for 4 years over the injection wells)

Vertical displacements at 3 years (left) and time evolution of vertical displacement for a location above KB501 (right) (Rutqvist et al., 2009)
The main CO2 storage aquifer (C10.2) is approximately 20–25m thick.

The C10.2 formation is overlain by a tight sandstone and siltstone formation (C10.3) of about 20m in thickness.

The C10 formation, together with the lower cap rock (C20.1–C20.3), form the CO2 storage complex at Krechba.

It has been shown that most of the observed uplift may be attributed to the poroelastic expansion of the 20m thick storage formation, but a significant contribution could come from pressure-induced deformation within a larger zone (~100m thick) of shale sands immediately above the injection zone (Rutqvist et al. (2009)).
Geomechanic Domain

Overburden

Reservoir

Underburden
Summary

- Dynamic flow data (BHP and CO2 saturation) and surface deformation very sensitive to geomechanical properties of the formation such as Young’s modulus and Poisson ratio. Reservoir traction an important source of uncertainty in injection and production data.

- Integration of geomechanical observed data in addition to flow data should be considered for better reservoir characterization.

- Future plan: Full field reservoir simulation and characterization of In Salah reservoir using observed data from three injection wells and surface uplift InSAR data.
Conclusions

• General hexahedral elements to handle complex reservoir geometries
• Full tensor permeability and dispersion
• Locally mass conservative and accurate flux description
• Reduced grid orientation effect on pressure and concentration
• Integration of single, two, black oil, and compositional formulations under a single MFMFE framework
• Extension to coupled ASP and/or compositional flow and geomechanics for fractured reservoirs
• Coupling with phase field for fracture propagation