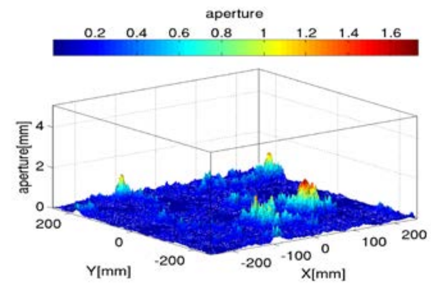
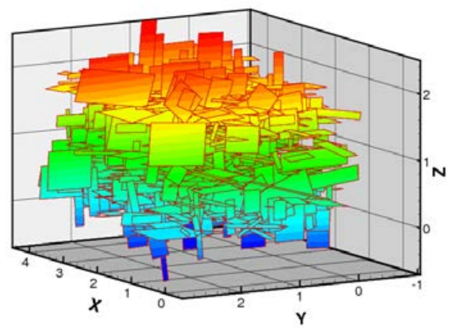
Modelling and Simulation in Porous Media



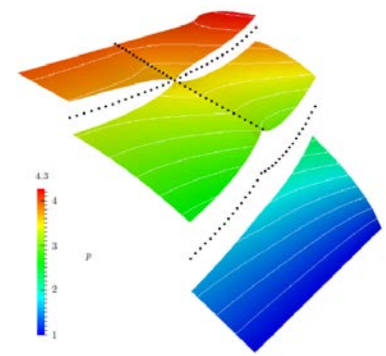
Jenny

Nui



Jean Roberts

Jérôme Jaffré



Markus

Nicolas

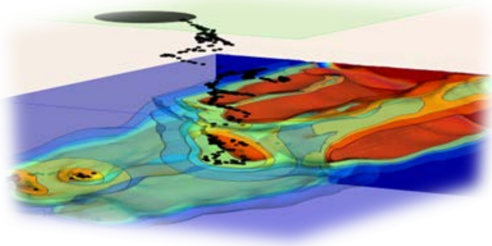
Multi-scale and multi-physics modelling of flow and transport processes in porous media

Beatrix Becker, Martin Schneider, Markus Wolff, Benjamin Faigle,
Bernd Flemisch, Rainer Helmig
University of Stuttgart, Germany

In co-operation with
Ivar Aavatsmark (Uni. Bergen), Hamdi Tchelepi (Stanford University)

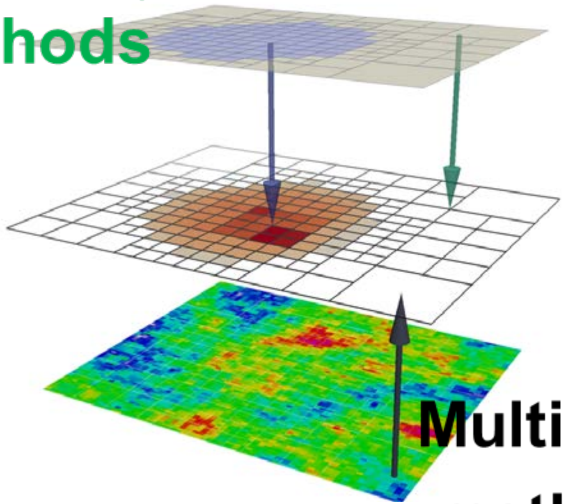
Need for Coupled Porous Media Models

**NAPL, …
contaminations**



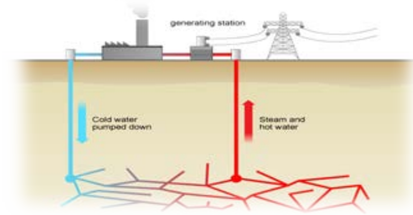
"Direct" ground water contamination?

Multi-physics methods



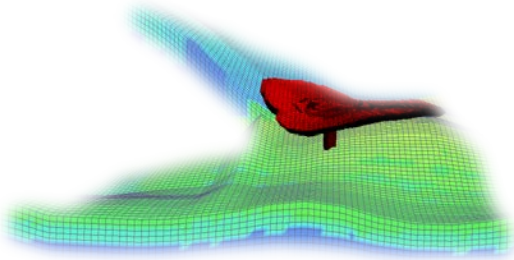
Multi-scale methods

Geothermal energy



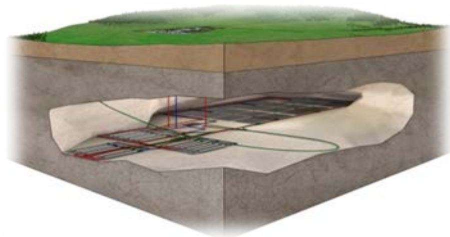
Fracking fluid and salt water infiltration in groundwater?

Carbon dioxide, methane storage



CO2, methane and salt water infiltration in groundwater?

Radioactive waste deposit

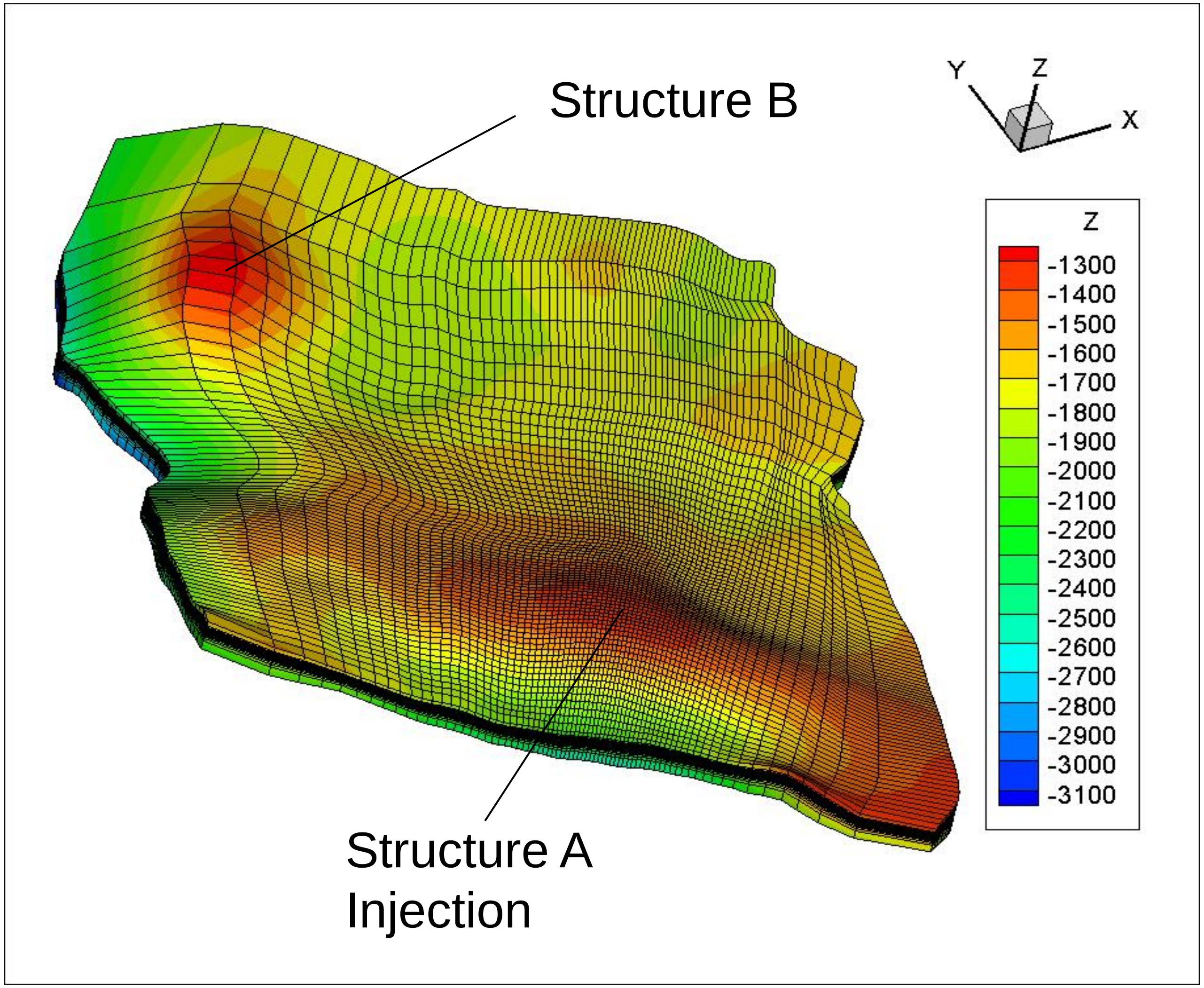


Radioactive contamination of groundwater?

Unconventional gas production

Fracking fluid and methane migration in groundwater?

Potential CO_2 storage site in northern Germany



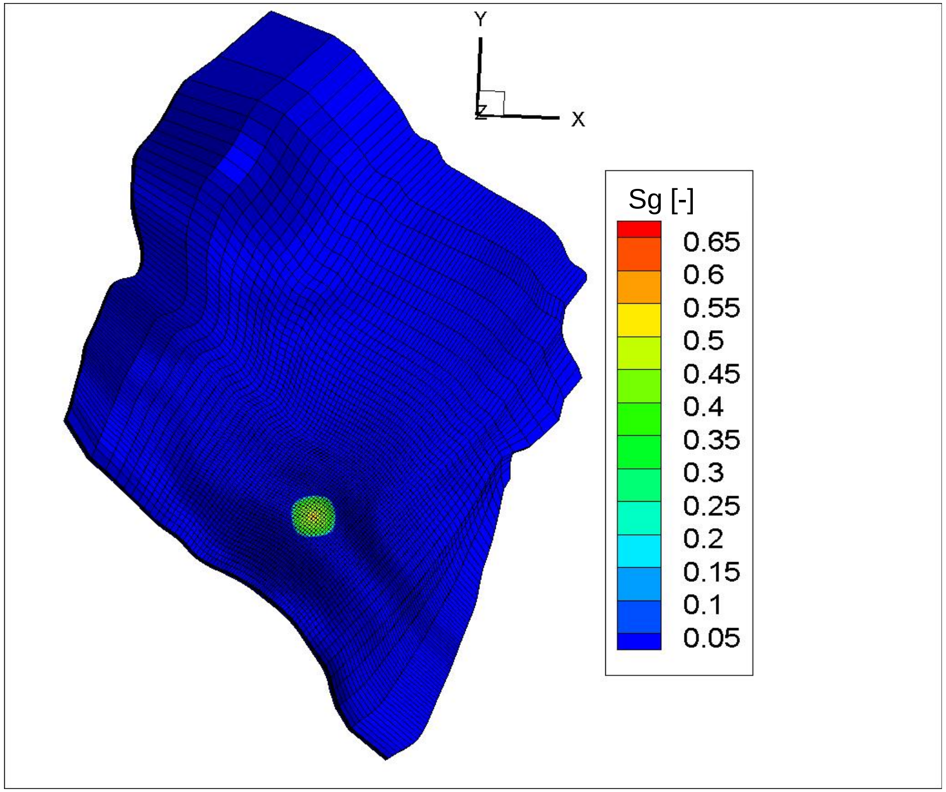
- 40 km x 40 km site in northern Germany
- Injection depth: 1500m
- Injection rate: 2.5 Mt/year (for 10 years)

How does the injection in structure A affect the pressure field in structure B?

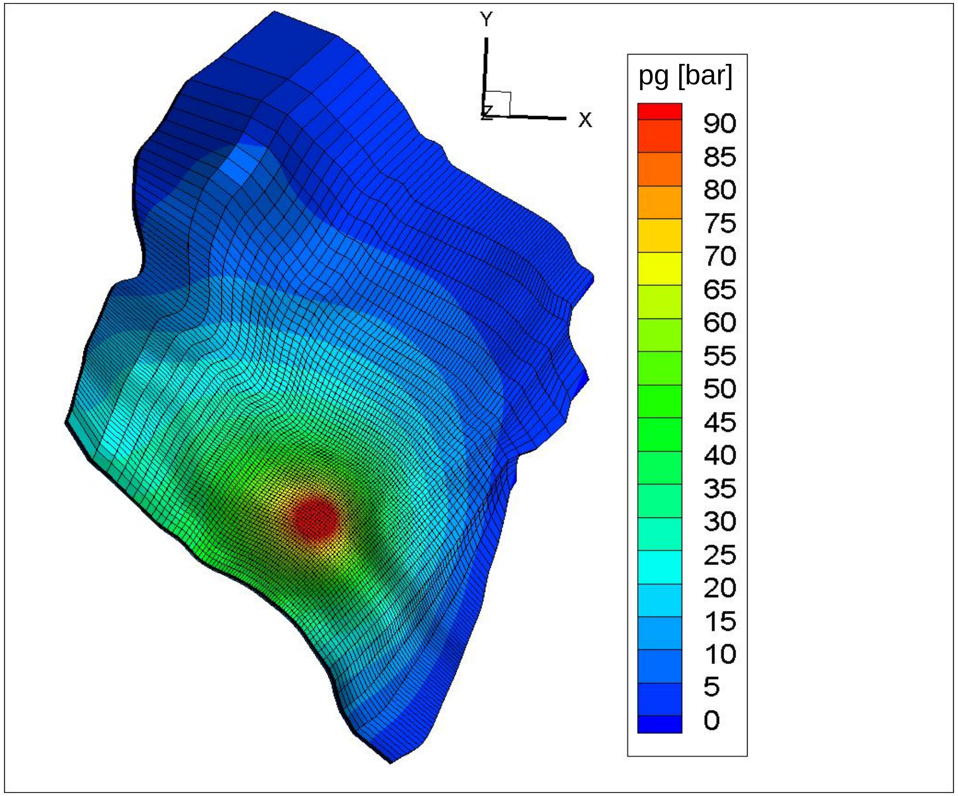
(Walter et.al 2010)

Saturation and pressure distribution after 10 years

CO_2 saturation

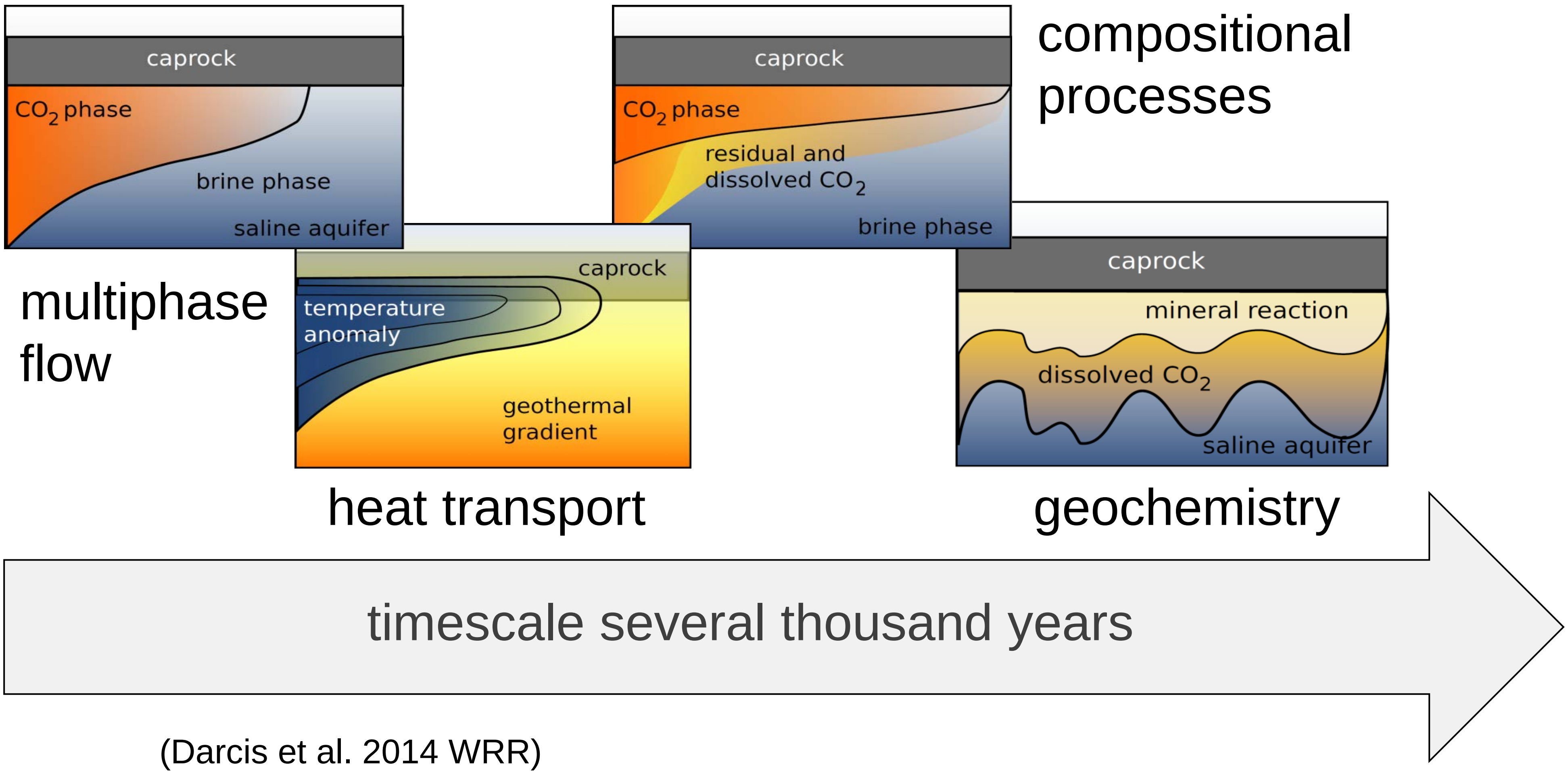


Pressure increase



Complex Long-Term Processes: e.g. CO_2 Sequestration:

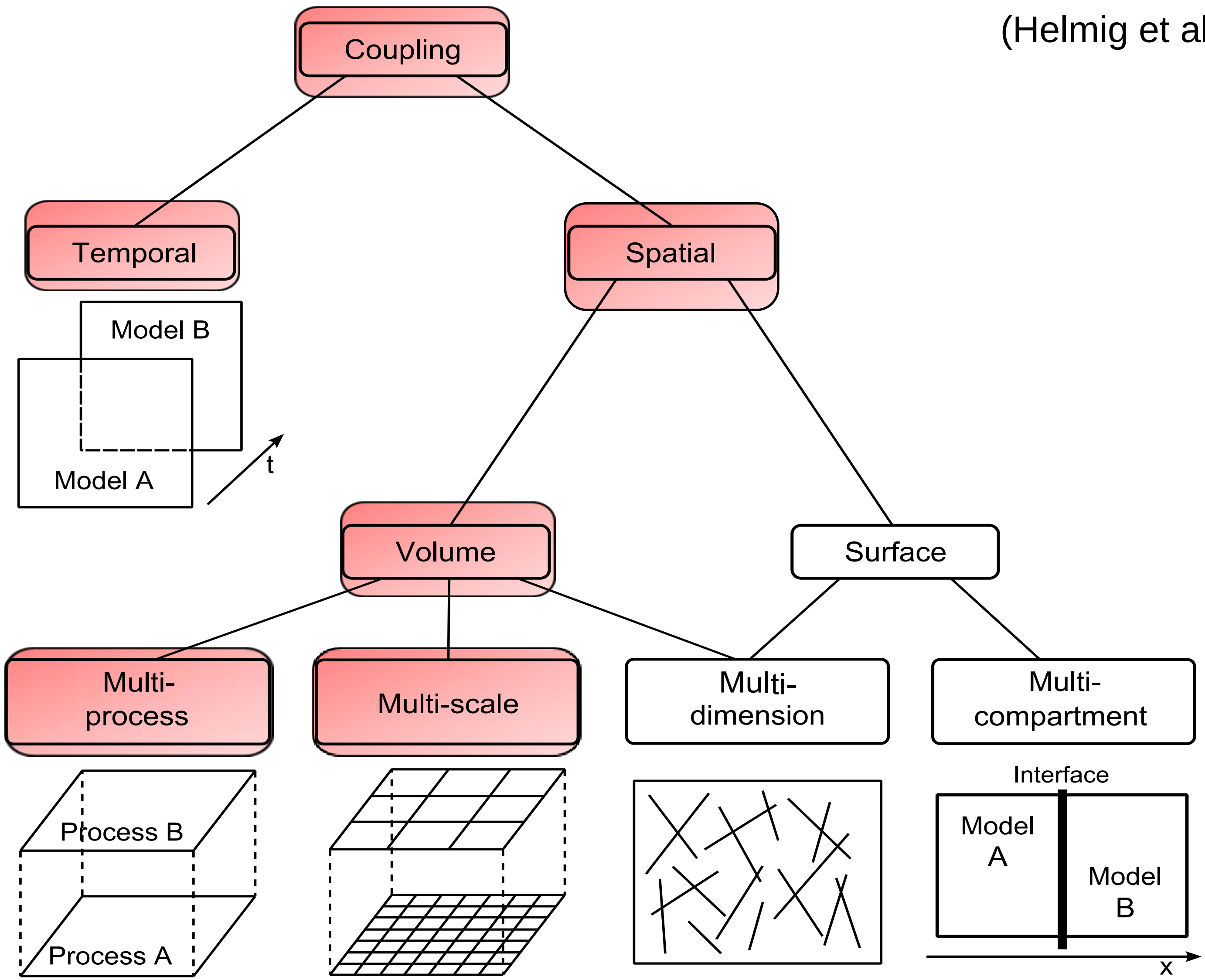
Simulation of long-term processes during CO_2 storage



(Darcis et al. 2014 WRR)

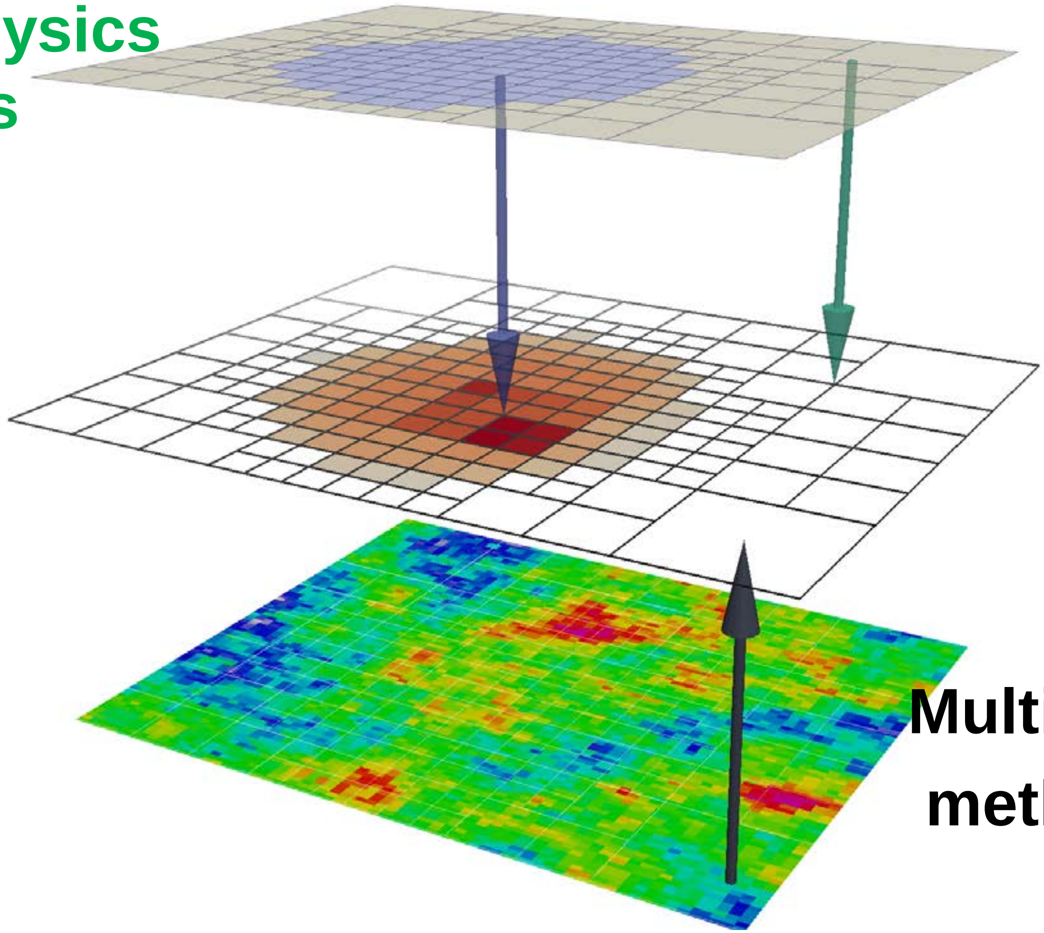
Classification of Model-Coupling Approaches

(Helmig et al. 2012 AWR)



Multi-scale and multi-physics strategy

Multi-physics methods

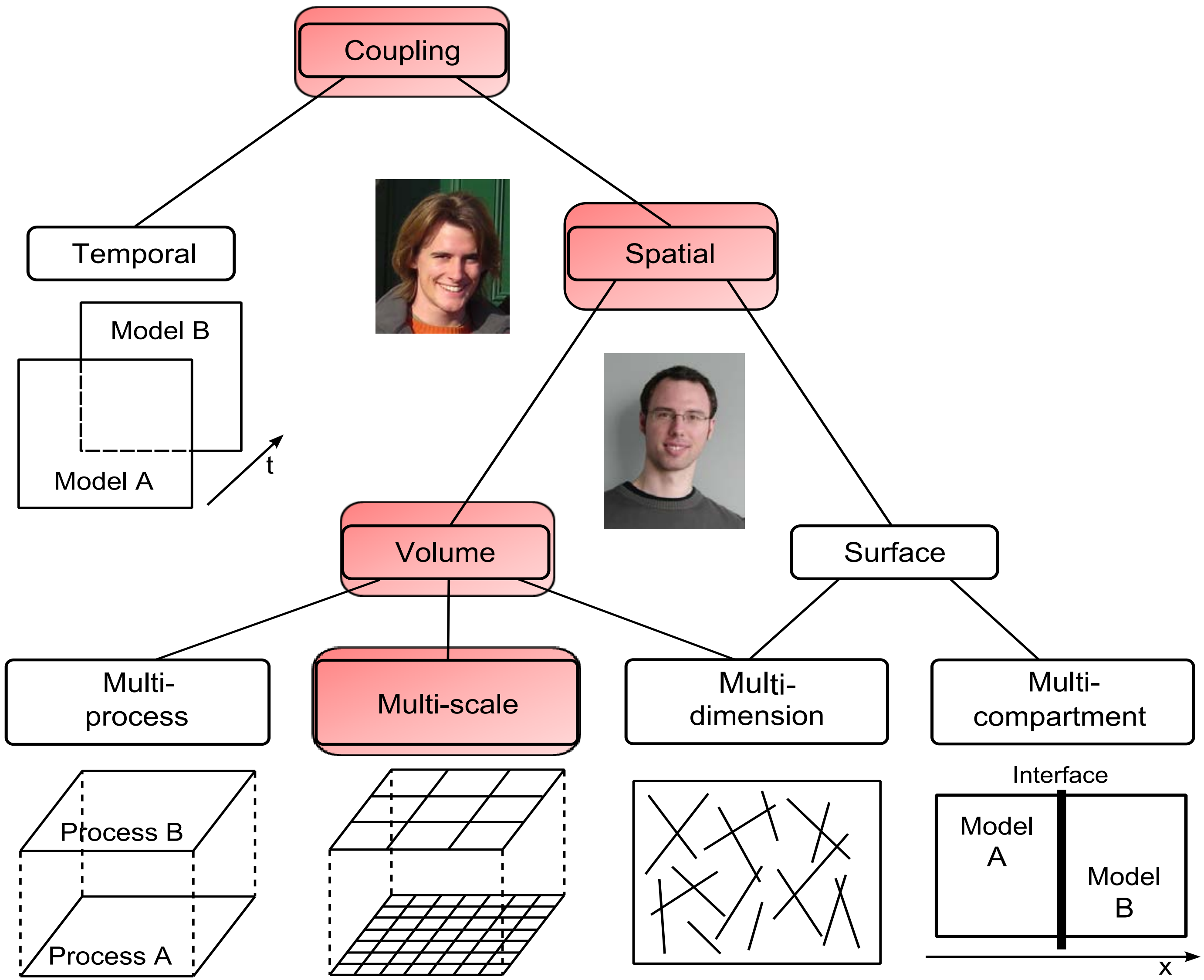


DuMuX

Multi-scale methods

adaptive in space (multi-scale), adaptive in physics (multi-physics) + adaptive in time (multi-numerics)

Multi-scale Approaches



Model equations: Equations for two-phase flow

- **Coupled system:**
  - Continuity equation: $$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{v}_\alpha = q_\alpha \qquad \alpha \in \{\mathrm{w}, \ \mathrm{n}\}$$
  - Darcy´s law: $$\mathbf{v}_\alpha = -\frac{k_{\mathrm{r}_\alpha} \mathbf{K}}{\mu_\alpha} \cdot [\nabla p_\alpha - \varrho_\alpha g \nabla z]$$

$$p_{\mathrm{c}} = p_{\mathrm{n}} - p_w$$

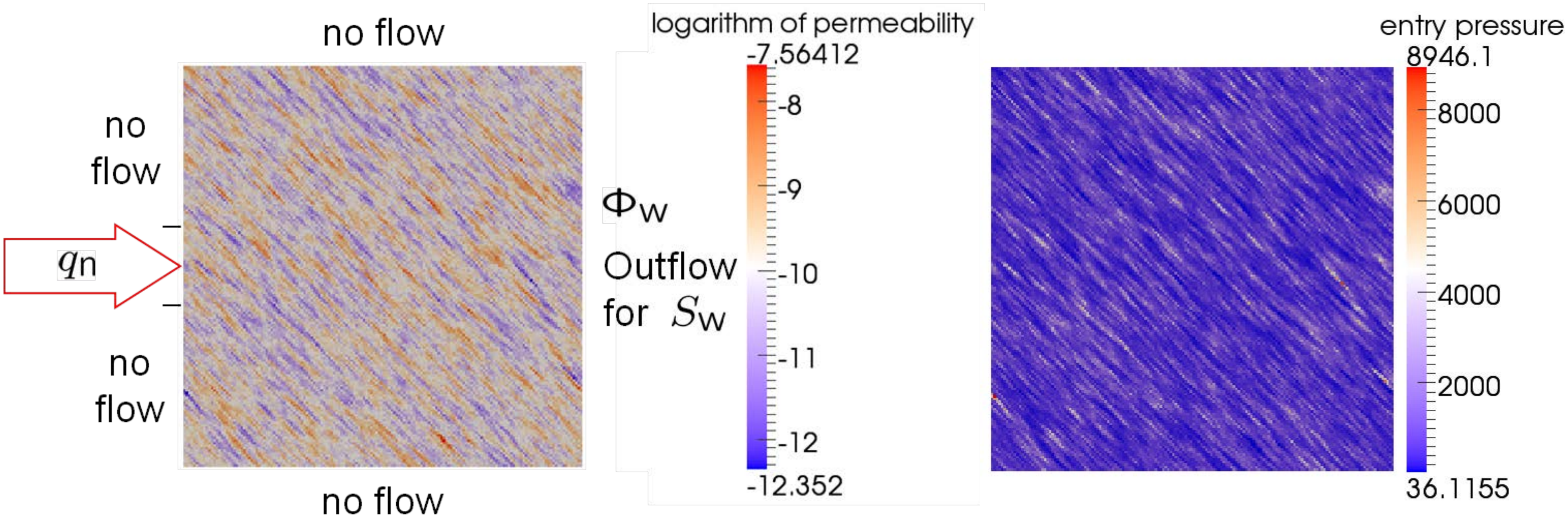
Demands on simulators

- Simulation of huge domains
- Fine resolution of heterogeneous parameters

Demands on simulators

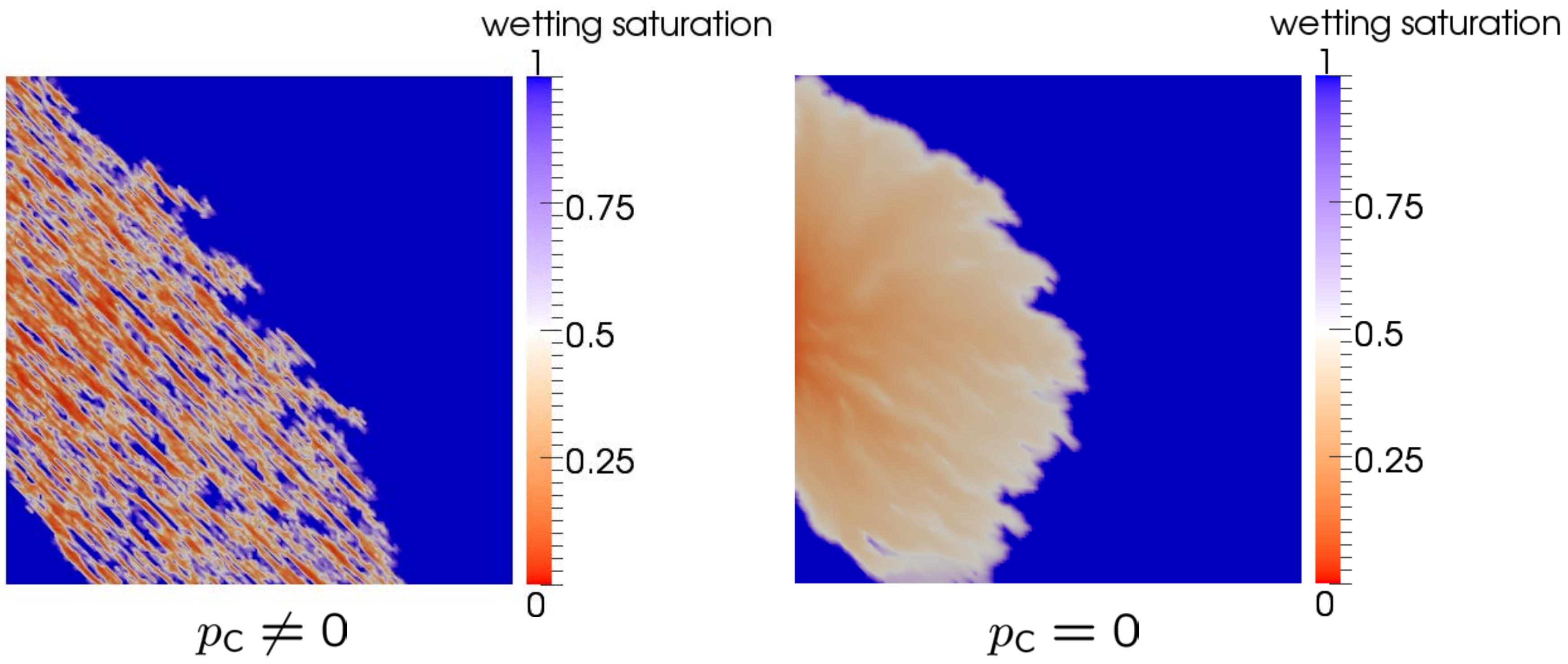
- Simulation of huge domains
- Fine resolution of heterogeneous parameters
- Fine resolution of fluid fronts
- Complex physics
  - Focus: e.g. Two-phase flow including **capillary pressure effects**

Some examples (Capillary pressure effects)



Fine-scale reference: Effect of capillary pressure

- Capillary pressure heterogeneities can strongly affect the fluid distribution!



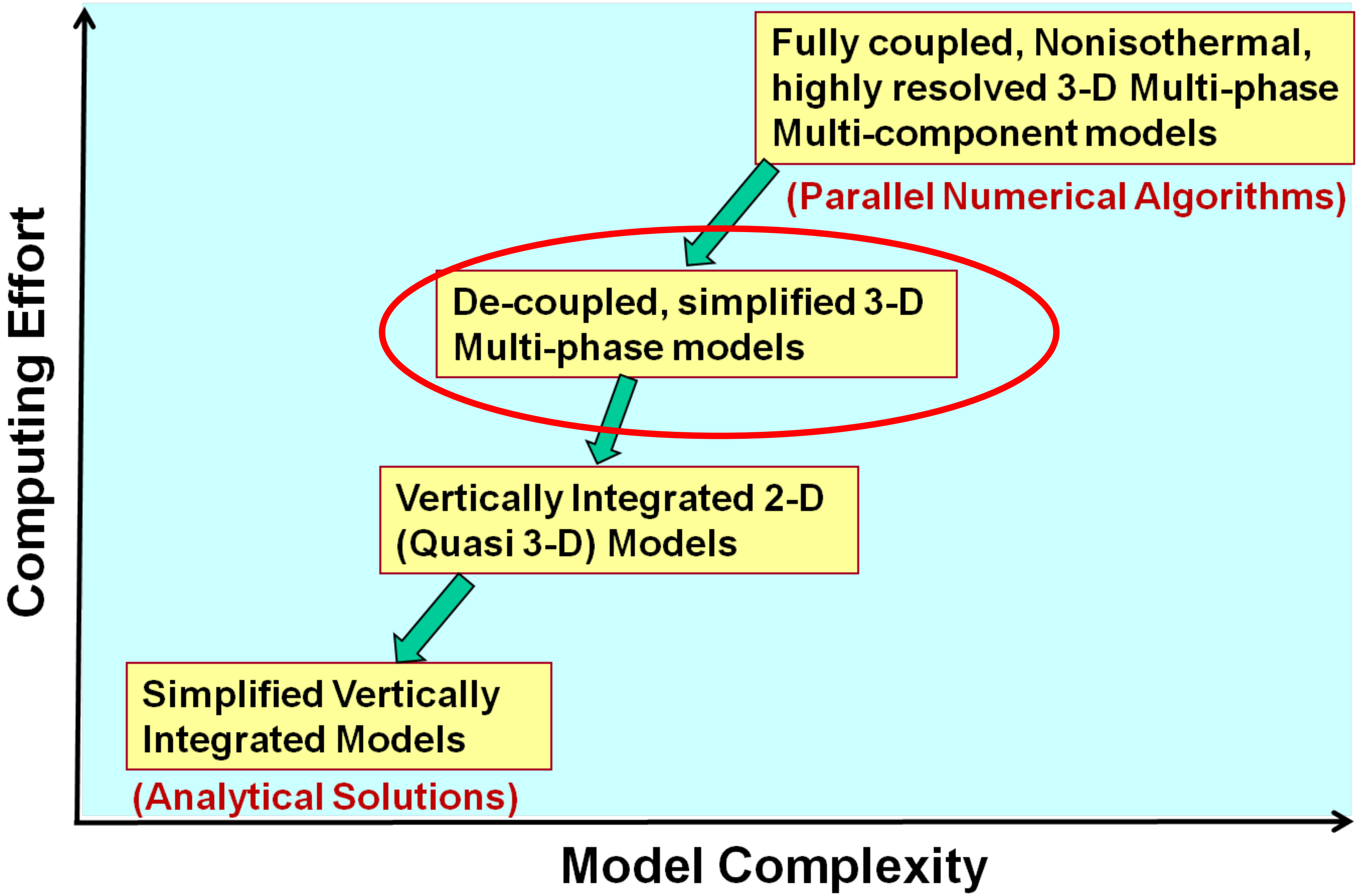
Demands on simulators

- Simulation of huge domains
- Fine resolution of heterogeneous parameters
- Fine resolution of fluid fronts
- Complex physics
  - Focus: Two-phase flow including **capillary pressure effects**

Problems:

➡ Large number of grid cells/
Large number of degrees of freedom

➡ Complex non-linear models


Computing Effort
Fully coupled, Nonisothermal, highly resolved 3-D Multi-phase Multi-component models
(Parallel Numerical Algorithms)
De-coupled, simplified 3-D Multi-phase models
Vertically Integrated 2-D (Quasi 3-D) Models
Simplified Vertically Integrated Models
(Analytical Solutions)
Model Complexity

(M. Celia)

Demands on simulators

- Simulation of huge domains
- Fine resolution of heterogeneous parameters
- Fine resolution of fluid fronts
- Complex physics
  - Focus: Two-phase flow including **capillary pressure effects**

Problem:

→ Large number of grid cells/ Large number of degrees of freedom

→ Complex non-linear systems/models

Solution Strategies:

Multi-scale methods

Simplified de-coupled systems

Multi-physics methods

Multi-scale modeling of two-phase flow: State of the art

- Many powerfull approaches for the simplified case:

$$\nabla \cdot [-\lambda_{\mathrm{t}} \mathbf{K} \nabla p_{\mathrm{w}}] = q_{\mathrm{t}}$$

$$\phi \frac{\partial S_{\mathrm{w}}}{\partial t} + \nabla \cdot \mathbf{v}_{\mathrm{w}} = q_{\mathrm{w}}$$

- Multi-scale finite element approaches (e.g. Kippe et al., 2008, …)
- Multi-scale finite volume approaches (e.g. Lee et al., 2009, …)
- Standard numerical upscaling/pseudo function approaches (e.g. Durlofsky,1991, Stone, 1991, Niessner etal 2009…)
- Adaptive upscaling methods (e.g. Chen and Li, 2009,…)
- …

Model equations: Equations for two-phase flow

- Coupled system:
  - Continuity equation: $$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{v}_\alpha = q_\alpha \qquad \alpha \in \{\mathrm{w},\ \mathrm{n}\}$$
  - Darcy´s law: $$\mathbf{v}_\alpha = -\frac{k_{\mathrm{r}_\alpha} \mathbf{K}}{\mu_\alpha} \cdot [\nabla p_\alpha - \varrho_\alpha g \nabla z]$$
- **Reformulation → Decoupled equations:**
  - Pressure equation:

$$\nabla \cdot [-\lambda_\mathrm{t} \mathbf{K} \nabla p_\mathrm{w} - \lambda_\mathrm{n} \mathbf{K} \nabla p_\mathrm{c} - (\lambda_\mathrm{w} \rho_\mathrm{w} + \lambda_\mathrm{n} \rho_\mathrm{n}) \mathbf{K}\, g \nabla z] = q_\mathrm{t}$$

$$p_\mathrm{c} = p_\mathrm{n} - p_w$$

  - Transport equation: $$\phi \frac{\partial S_\mathrm{w}}{\partial t} + \nabla \cdot \mathbf{v}_\mathrm{w} = q_\mathrm{w}$$

Multi-scale modeling modeling of two-phase flow

→ including capillary pressure

$$\nabla \cdot [-\lambda_t \mathbf{K} \nabla p_w] = q_t$$

⬇

$$\nabla \cdot [-\lambda_t \mathbf{K} \nabla p_w - \lambda_n \mathbf{K} \nabla p_c - (\lambda_w \rho_w + \lambda_n \rho_n) \mathbf{K}\, g \nabla z] = q_t$$

Outline part I

- Numerical upscaling
- Adaptive grid refinement
- Multi-scale modeling

Numerical Upscaling

- **Coarse scale equations**:
  - Mass balance: $$\phi^* \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{v}_\alpha = q_\alpha \qquad \alpha \in \{\text{w, n}\}$$
  - Darcy´s law: $$\mathbf{v}_\alpha = -\frac{\mathbf{K}^*_{\text{tot}_\alpha}}{\mu_\alpha} \cdot [\nabla p_\alpha - \varrho_\alpha g \nabla z]$$
- Choose representative **subscale problems** (problem setups)
- Calculate **coarse scale quantities** from (local) fine-scale simulations

Numerical Upscaling

- Coarse scale equations:
  - Mass balance:

$$\phi^* \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{v}_\alpha = q_\alpha \qquad \alpha \in \{\mathrm{w},\ \mathrm{n}\}$$

  - Darcy´s law:

$$\mathbf{v}_\alpha = -\frac{\mathbf{K}^*_{\mathrm{r}_\alpha}\mathbf{K}^*}{\mu_\alpha} \cdot [\nabla p_\alpha - \varrho_\alpha g \nabla z]$$

- **Reformulation:**
  - Pressure equation:

$$\nabla \cdot [-\mathbf{\Lambda}^*_{\mathrm{t}}\mathbf{K}^*\nabla\Phi_{\mathrm{w}} - \mathbf{F}^*_{\mathrm{n}}\mathbf{\Lambda}^*_{\mathrm{t}}\mathbf{K}^*\nabla\Phi^*_{\mathrm{c}}] = q_{\mathrm{t}}$$

$$\Phi_\alpha = p_\alpha + \varrho_\alpha g z \qquad \Phi_{\mathrm{c}} = \Phi_{\mathrm{n}} - \Phi_w$$

  - Transport equation:

$$\phi^* \frac{\partial S_{\mathrm{w}}}{\partial t} + \nabla \cdot \mathbf{v}_{\mathrm{w}} = q_{\mathrm{w}}$$

(Wolff et al. Treatment of tensorial relative permeabilities with multipoint flux approximation. *International journal of numerical analysis & modeling*, 2012)

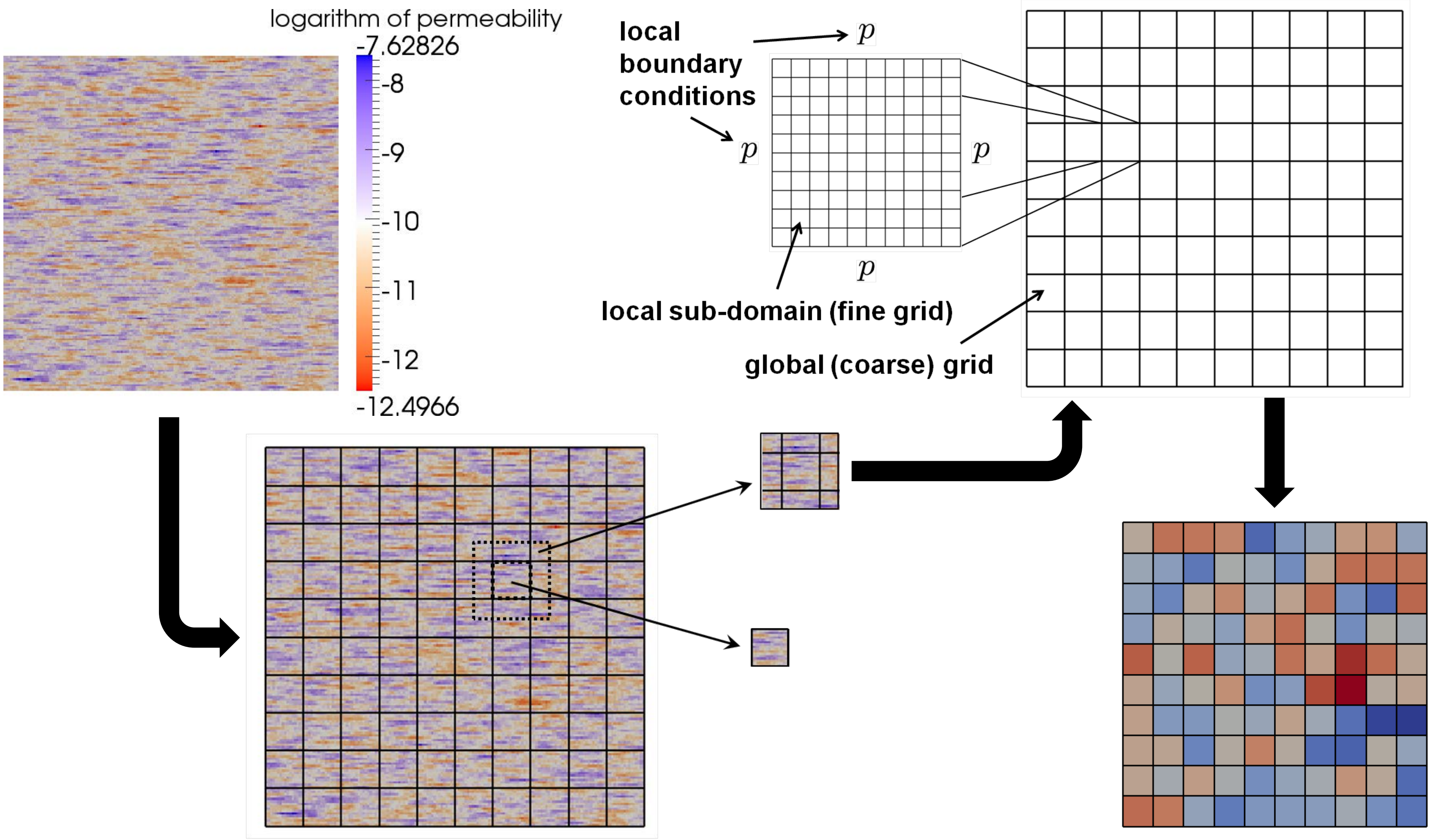
Definition of the phase permeability tensor

$$\mathbf{K}^*_{\mathsf{tot}_\alpha} = \mathbf{K}^*\mathbf{K}^*_{\mathsf{r}_\alpha}$$

- **Total** and **absolute permeability:**
  - Second order tensors
- **Relative permeability:**
  - **Second order tensor** (e.g. *Nordbotten et al.* On the definition of macroscale pressure for multiphase flow in porous media, 2008)

$$\mathbf{K}^*_{\mathsf{tot}} = \begin{pmatrix} k^*_{xx} & k^*_{xy} & k^*_{xz} \\ k^*_{yx} & k^*_{yy} & k^*_{yz} \\ k^*_{zx} & k^*_{zy} & k^*_{zz} \end{pmatrix} \begin{pmatrix} k^*_{\mathsf{r}_{xx}} & k^*_{\mathsf{r}_{xy}} & k^*_{\mathsf{r}_{xz}} \\ k^*_{\mathsf{r}_{yx}} & k^*_{\mathsf{r}_{yy}} & k^*_{\mathsf{r}_{yz}} \\ k^*_{\mathsf{r}_{zx}} & k^*_{\mathsf{r}_{zy}} & k^*_{\mathsf{r}_{zz}} \end{pmatrix} \quad \text{(3-D)}$$

Local numerical upscaling



Local steady state upscaling

- Calculate full **absolute permeability** tensor from **local 1-p problems**
  - e.g. Wen et al. 2003, …

Local steady state upscaling

- Calculate full **absolute permeability** tensor from **local 1-p problems**
  - e.g. Wen et al. 2003, …
- Calculate **capillary pressure** using a **macroscale percolation approach**
  - e.g. Braun et al 2005, ….

Local steady state upscaling:

Assumption:
local capillary equilibrium

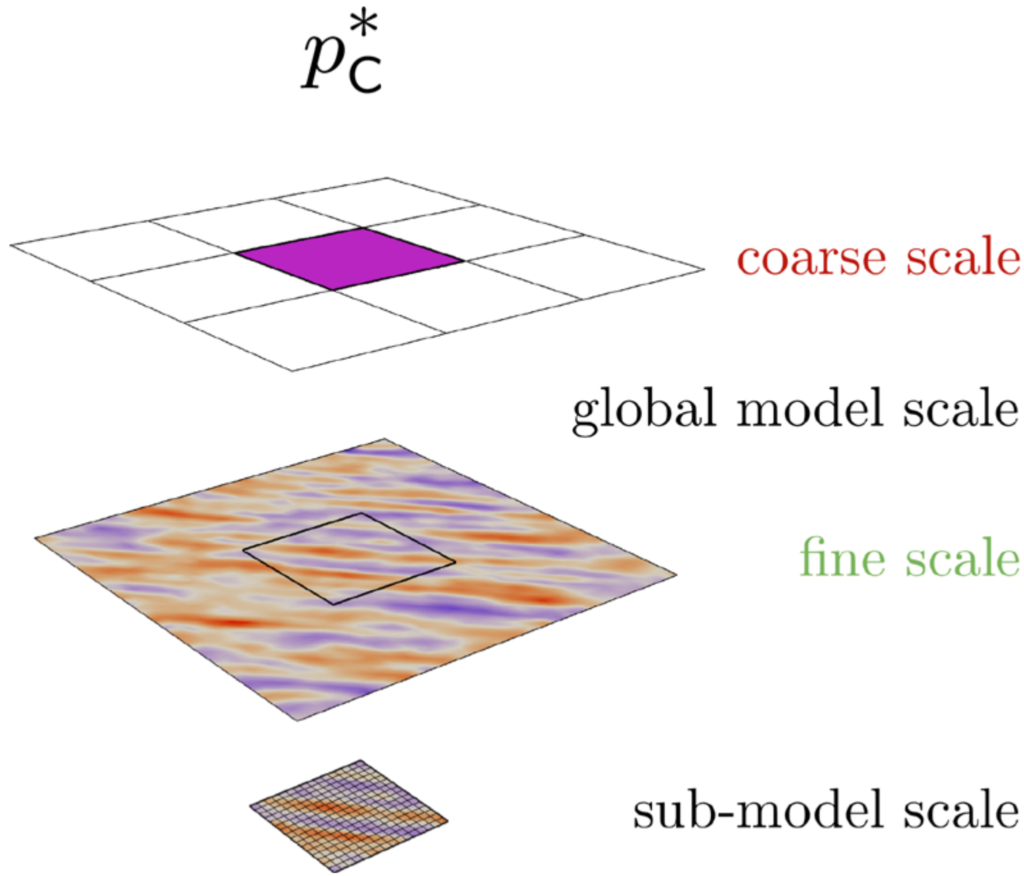
- Perculation approach

Saturation distribution

- Averaging

$$p_c^* = p_n^* - p_w^*$$

Capillary pressure



(Braun et al. JCH 2005, Nuske et al WRR, 2010)

Local steady state upscaling

- Calculate full **absolute permeability** tensor from **local 1-p problems**
  - e.g. Wen et al. 2003, …
- **Calculate capillary** pressure using a **macroscale percolation approach**
  - e.g. Braun et al 2005, ….
- Calculate full **relative permeability** tensor from **local 2-p problems**
  - Extention of Wen et al. 2003 for relative permeabilities

Local steady state upscaling:

Assumption: local capillary equilibrium

- Perculation approach

↓

Saturation distribution

- Solve local two-phase flow problem

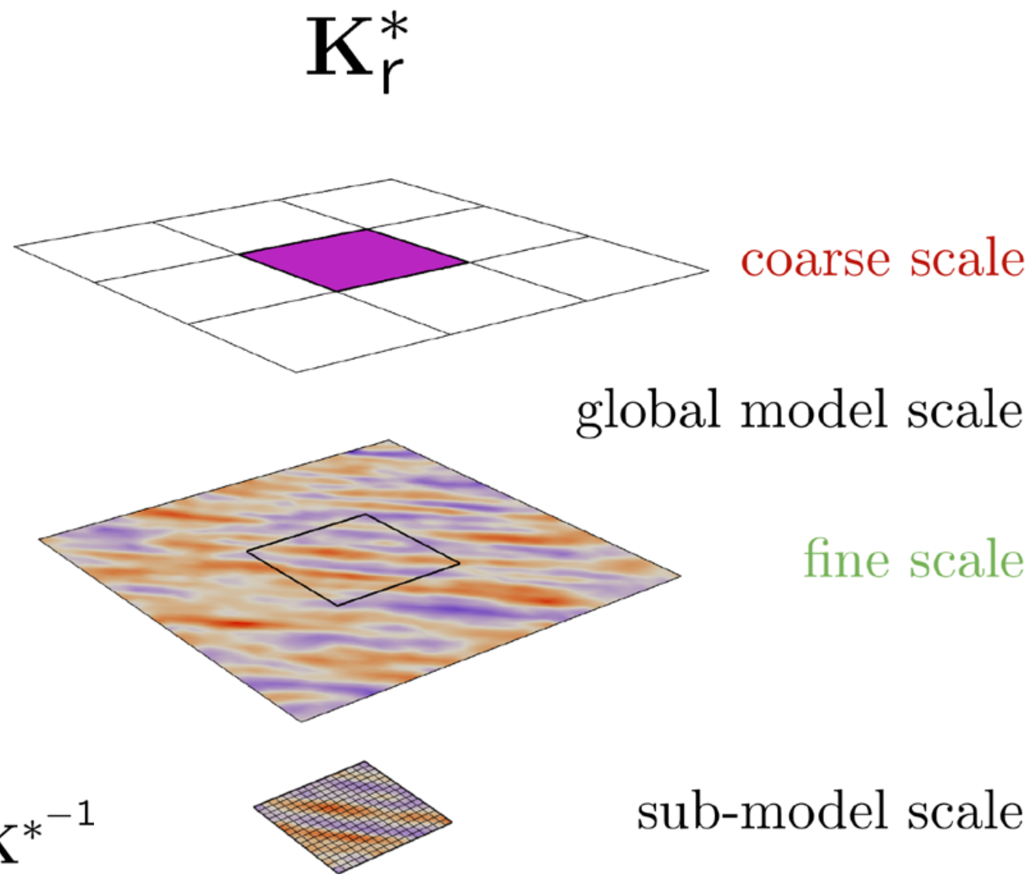
↓

Flow field

- Averaging

↓ $\mathbf{K}_r^* = \mathbf{K}_{tot}^* {\mathbf{K}^*}^{-1}$

Relative permeability tensor



Relative permeability upscaling

$$\begin{pmatrix} \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} \\ \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} K^*_{\text{tot}_{xx\alpha}} \\ K^*_{\text{tot}_{xy\alpha}} \\ K^*_{\text{tot}_{yx\alpha}} \\ K^*_{\text{tot}_{yy\alpha}} \end{pmatrix} = - \begin{pmatrix} \langle v_{\alpha_x} \rangle^x_\alpha \\ \left\langle v_{\alpha_y} \right\rangle^x_\alpha \\ \langle v_{\alpha_x} \rangle^y_\alpha \\ \left\langle v_{\alpha_y} \right\rangle^y_\alpha \\ 0 \end{pmatrix}$$

$$\Psi^x_{\alpha,x} = \frac{1}{\mu_\alpha} \frac{\partial}{\partial x} \langle \Phi_\alpha \rangle^x_\alpha \qquad \Psi^x_{\alpha,y} = \frac{1}{\mu_\alpha} \frac{\partial}{\partial y} \langle \Phi_\alpha \rangle^x_\alpha$$

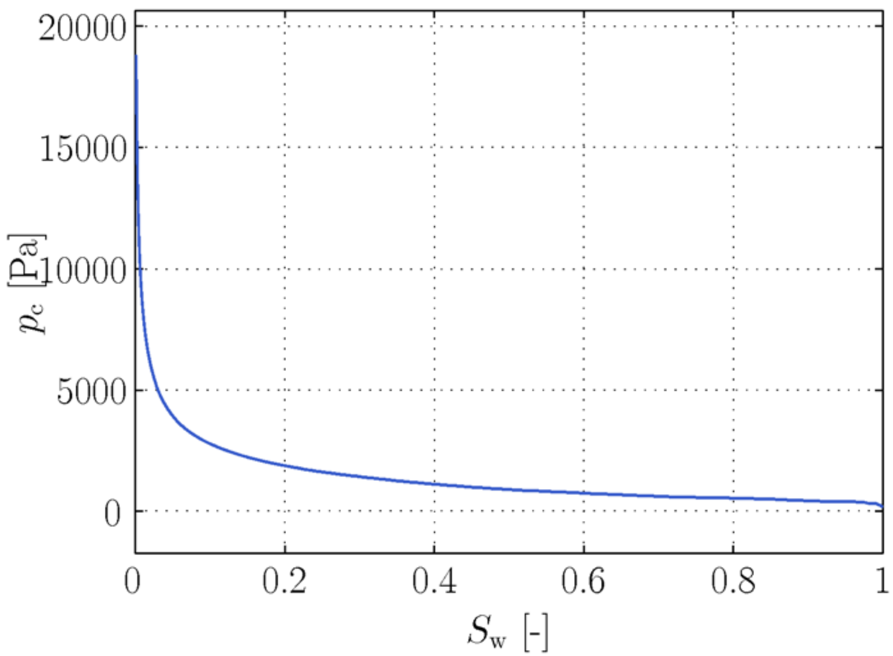
$$\Psi^y_{\alpha,x} = \frac{1}{\mu_\alpha} \frac{\partial}{\partial x} \langle \Phi_\alpha \rangle^y_\alpha \qquad \Psi^y_{\alpha,y} = \frac{1}{\mu_\alpha} \frac{\partial}{\partial y} \langle \Phi_\alpha \rangle^y_\alpha$$

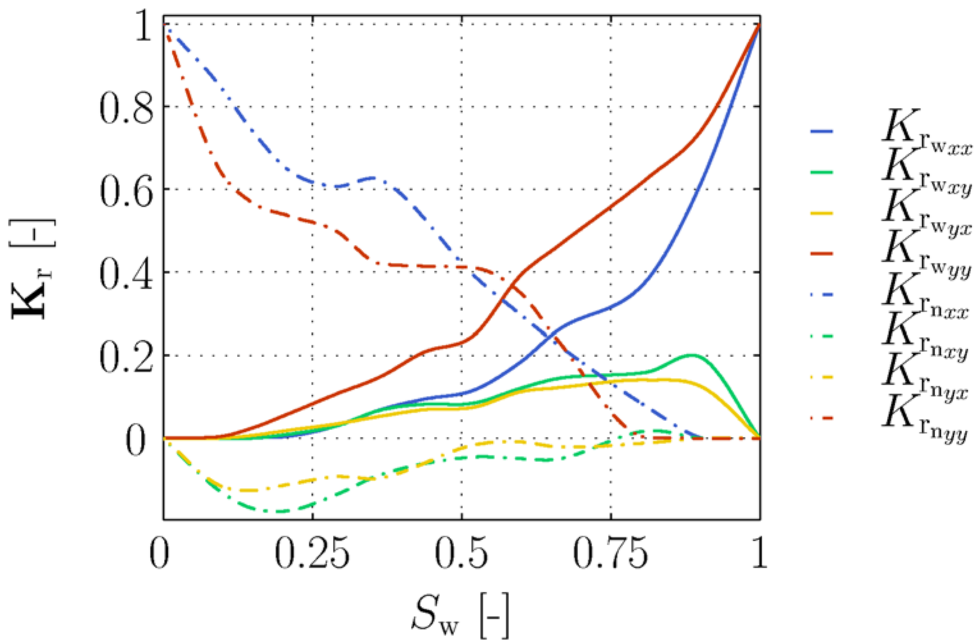
$$\mathbf{K}^*_r = \mathbf{K}^*_{\text{tot}} {\mathbf{K}^*}^{-1}$$

Local steady state upscaling

- Calculate full **absolute permeability** tensor from **local 1-p problems (**e.g. Wen et al. 2003, …)
- **Calculate capillary** pressure using a **macroscale percolation approach (**e.g. Braun et al 2005, ….)
- Calculate full **relative permeability** tensor from **local 2-p problems (**Extention of Wen et al. 2003 for relative permeabilities)
- Use **effective flux boundary conditions** to account for the global distribution of the heterogeneous parameters
- Permeability tensors are forced to be **symmetric** and **positive definit** **(**Avoid unphysical flow!)
- **Continuous relative permeability** curves are interpolated using **monotone splines**

Upscaled constitutive relations





Multi-scale = Upscaling + Downscaling

- **Upscaling:**
  - From detailed information to less detailed information
  - Information is thrown away
  - **Unique!**
    → A certain distribution has exact one average (unless the average operator is changed)
- **Downscaling:**
  - From less detailed information to detailed information
  - Information has to be generated!
  - **Non-unique!**
    → What is the distribution to a certain average?

Downscaling of two-phase flow

- Small scale phase pressures and saturations have to be **reconstructed from coarse scale information**
- Problem 1: Phase pressure and saturation are **coupled by capillary pressure**
- Problem 2: Information about **extreme values** is **lost/averaged** at the coarse scale

➡ Local downscaling is not possible if capillary effects are important!?

➡ Global downscaling is not efficient!

➡ **An adaptive grid is "a natural and efficient global downscaling strategy"!**

Adaptive grid refinement

Numerical method:

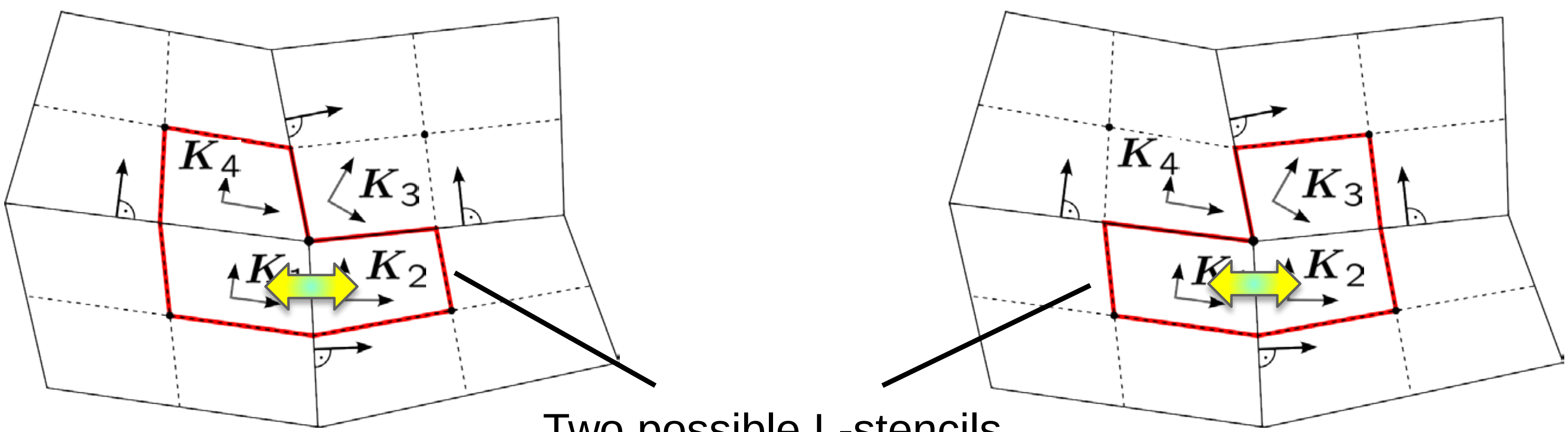
– Cell centered finite volumes with multi-point flux approximation (MPFA L-method, e. g. Aavatsmark et al., 2008.)

➡ Development of a MPFA L-method for two-phase flow including capillarity and gravity based on the decoupled formulation

- ✓ Non-conforming refinement with hanging nodes (Faigle et al. CompGeo 2013)
- ✓ Unstructured grids
- – Heterogeneities (e.g. Helmig and Huber, AWR 1999)
  - ✓ Permeability, Porosity
  - ✓ Capillary pressure (interface conditions)

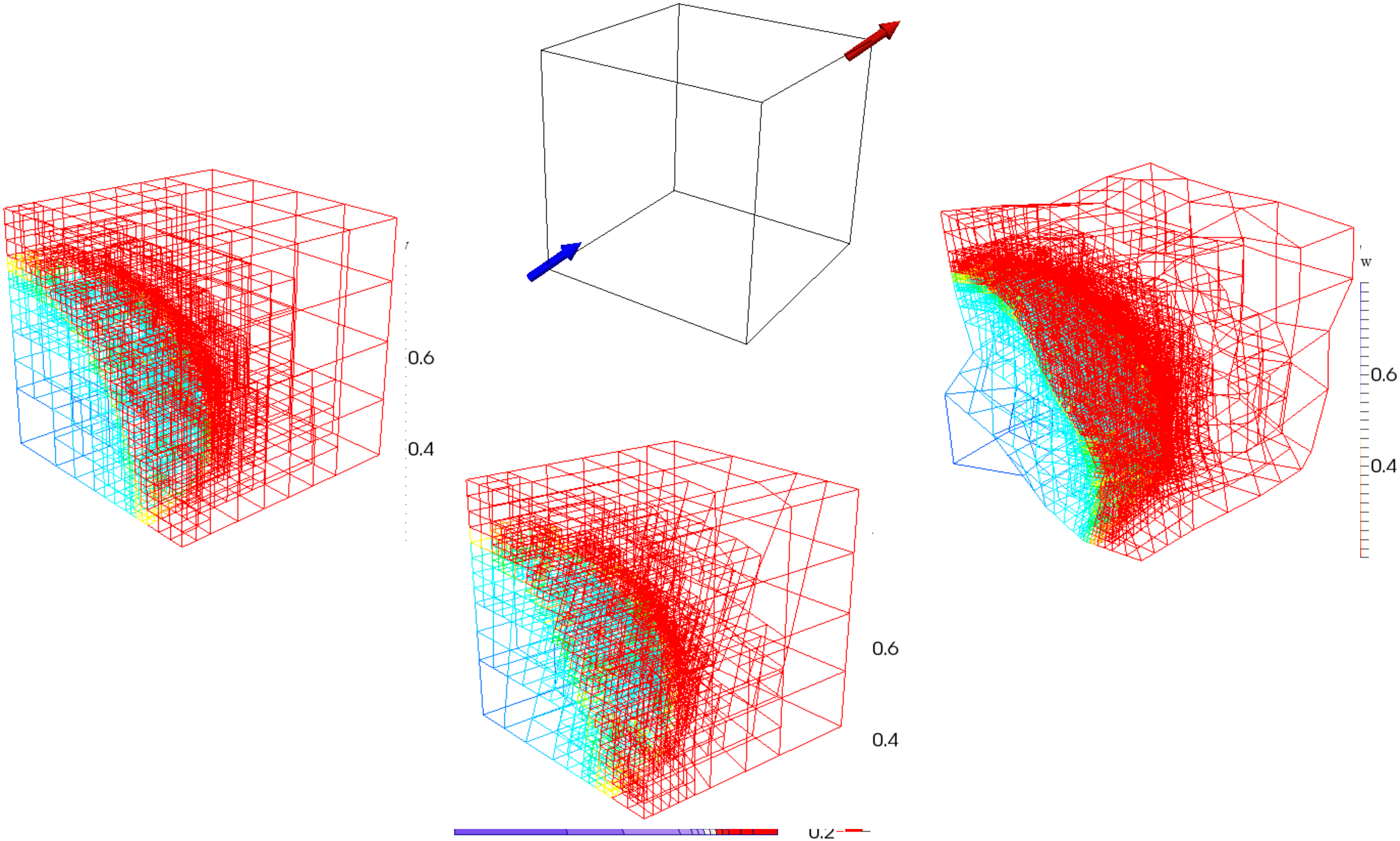
The MPFA L-method

- 2d-quadrilaterals: *Aavatsmark et al., Numer Meth Part D E, 2008*
- Works on **unstructured** and **non-*K*-orthogonal** grids
- Correct treatment of grid block heterogeneities and **material interfaces**
- Maximum flux stencil: 9-point stencil (2-d), 18-point stencil (3d)

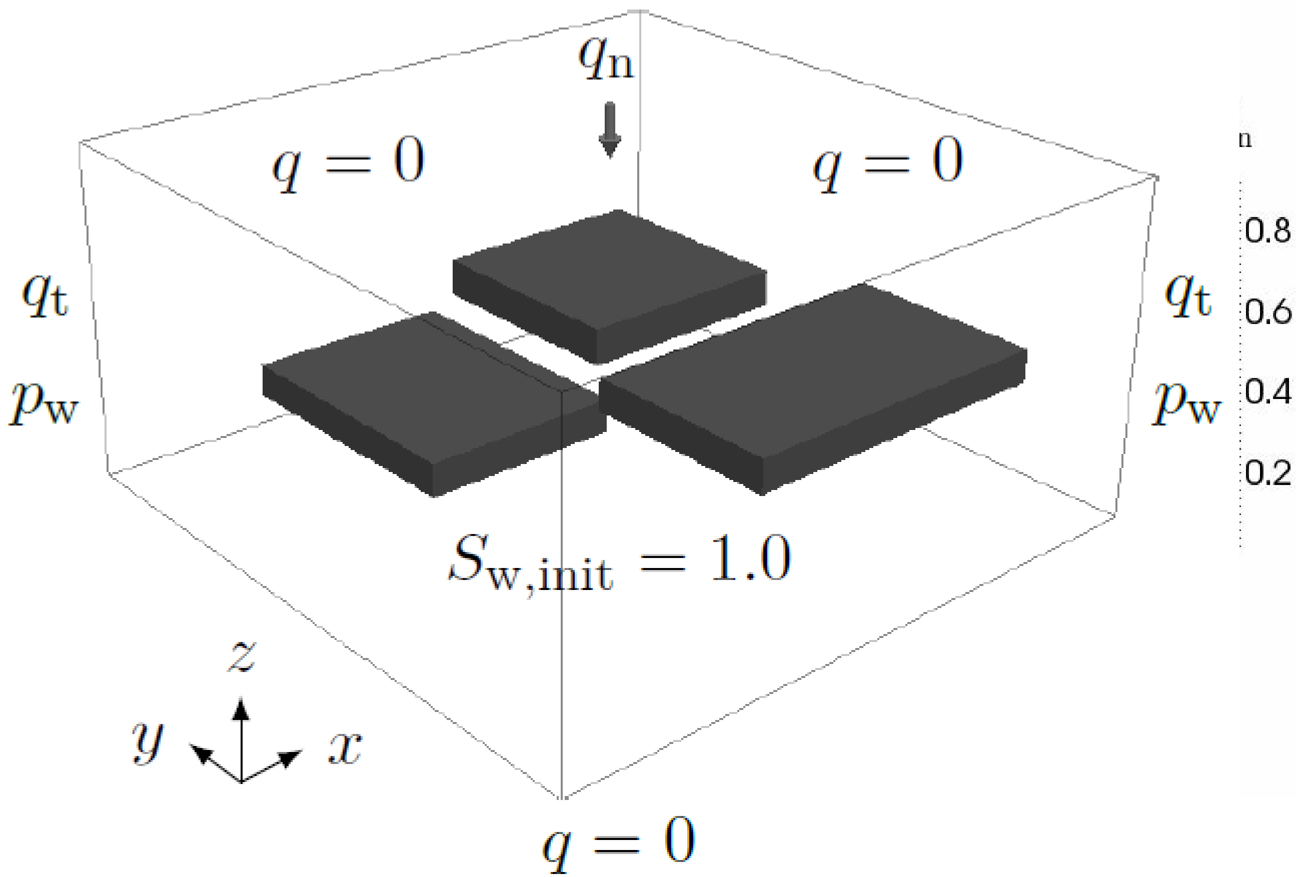


Two possible L-stencils
for flux approximation

Adaptation examples: 1) Nine-spot water flood

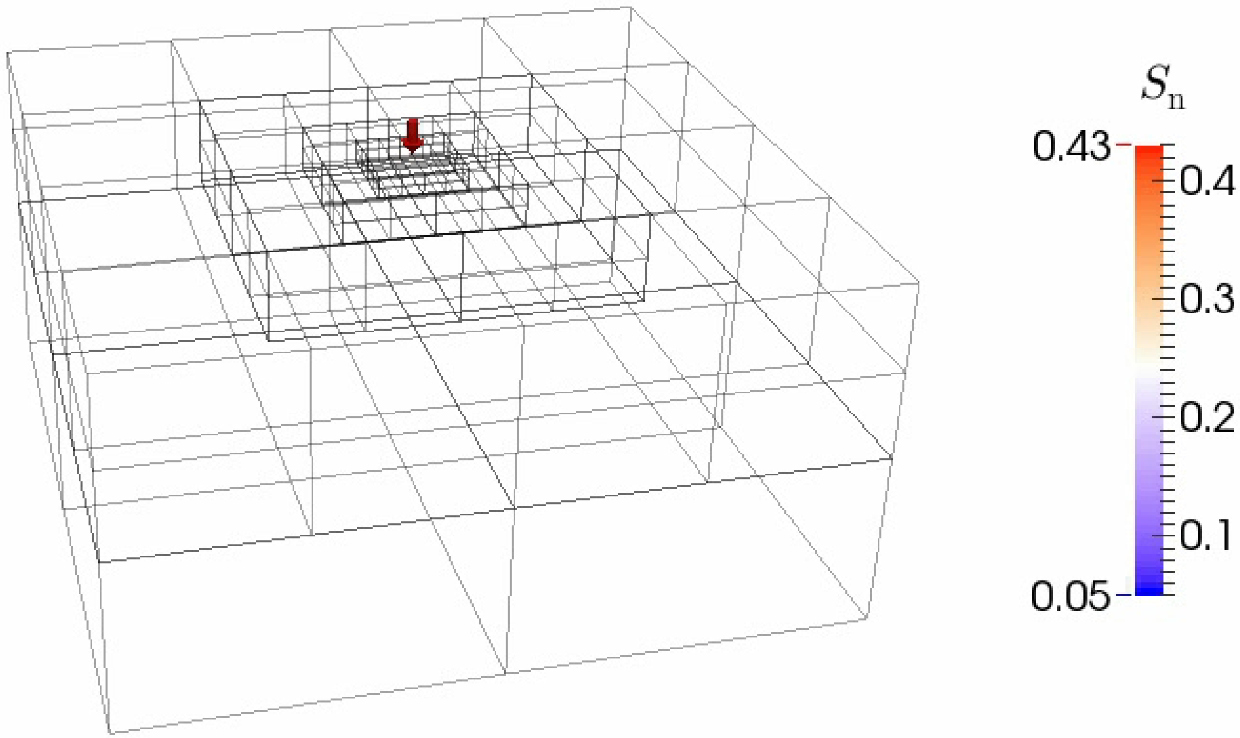


Adaptation examples: 2) Low permeable lenses



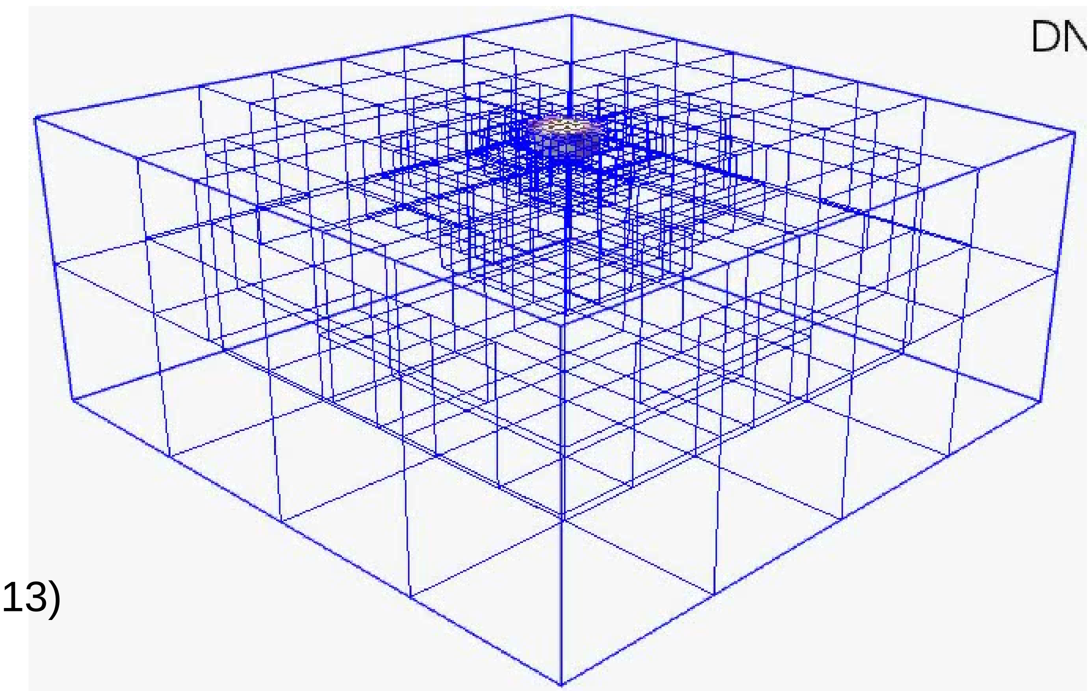
Adaptation examples: 3) Anisotropic permeability

$$\boldsymbol{K} = \begin{pmatrix} 10^{-10} & 0 & -5\times 10^{-11} \\ 0 & 10^{-10} & 5\times 10^{-11} \\ -5\times 10^{-11} & 5\times 10^{-11} & 5\times 10^{-11} \end{pmatrix} \ \mathrm{m}^2$$



Adaptive grid refinement: Example (DNAPL infiltration)

- Two-phase flow with capillary pressure and gravity
- Homogeneous domain, anisotrop absolute and relative permeability

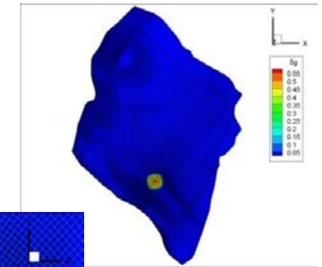


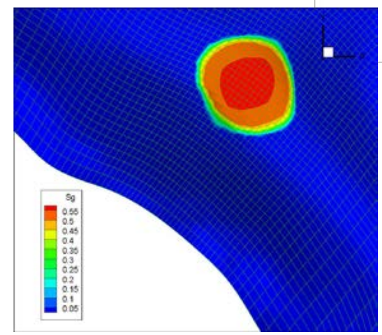
(Wolff et al. sub. DeGrutyer 2013)

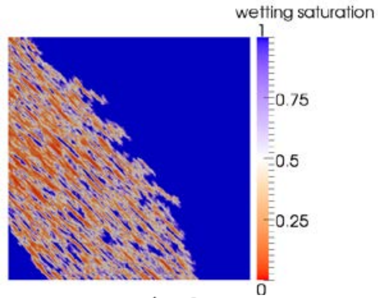
Multi-Scale Modeling

Demands on simulators:

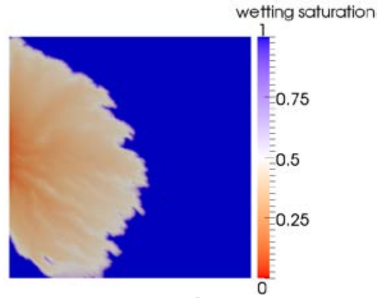
- Simulation of huge domains
- Fine resolution of heterogeneous parameters
- Fine resolution of fluid fronts
- Complex physics
  - Focus: Two-phase flow including **capillary pressure effects**



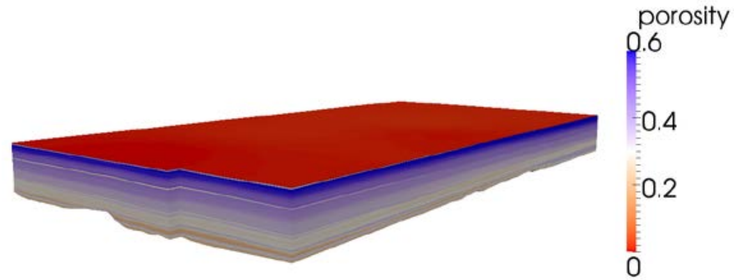




$p_c \neq 0$



$p_c = 0$



Multi-scale approach

- **Combination of numerical upscaling and grid adaptive discretization schemes** (Wolff et al. WRR 2013)
  - If a grid cell is on the finest level and fine-scale parameters/functions are available use these
  - Else use upscaled parameters/functions
- **Control** of the multi-scale behavior **by choice of the adaption criterion**!
  - Error control by **standard criteria**
    - e.g. saturation gradients, flux integrals, etc.
  - Error control by **multi-scale criteria**
    - Check if assumptions of the upscaling method are sufficiently satisfied (e.g. capillary equilibrium assumption → capillary number, etc.)

(Criteria)

- Standard

$$\text{mark element for} \begin{cases} \text{coarsening} & , \Delta S_{\text{local}} < \epsilon_{\text{coarsen}} \Delta S_{\text{max}} \\ \text{refinement} & , \Delta S_{\text{local}} > \epsilon_{\text{refine}} \Delta S_{\text{max}} \\ \text{nothing} & , \text{else} \end{cases}$$

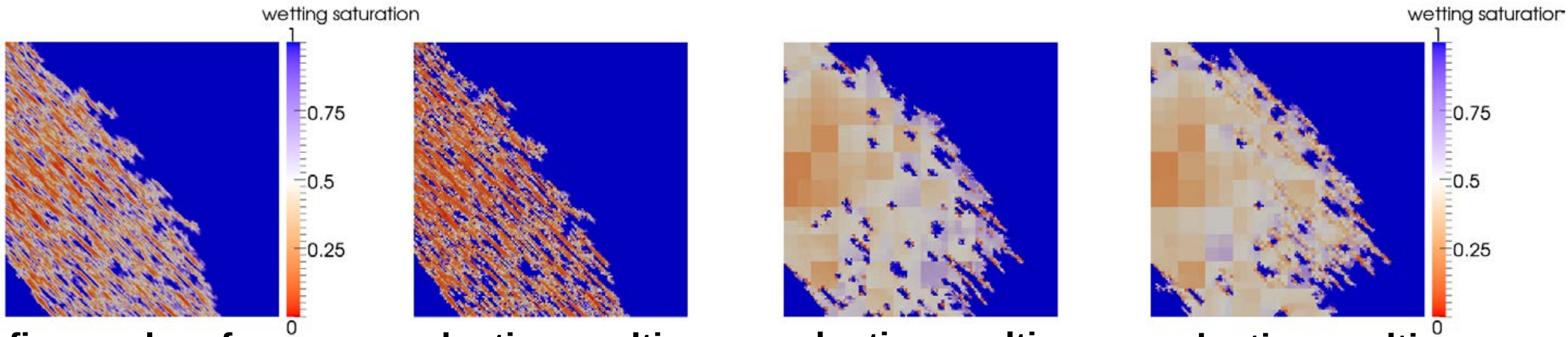
- Standard + ds/dt → flux integral!
  - additional coarsening criterion

$$\text{mark element for} \begin{cases} \text{coarsening} & , \left(\frac{\Delta S}{\Delta t}\right)_{\text{local}} < \epsilon_{\text{coarsen}} \end{cases}$$

- Standard + multi-scale (only with capillary pressure):
  - Check if capillary equilibrium assumption for coarse scale parameters is valid

$$\text{mark element for} \begin{cases} \text{coarsening} & , Ca_{\text{local}} < \epsilon_{\text{coarsen}} \\ \text{refinement} & , Ca_{\text{local}} > \epsilon_{\text{refine}} \end{cases}$$

Multi-scale: Random permeability field

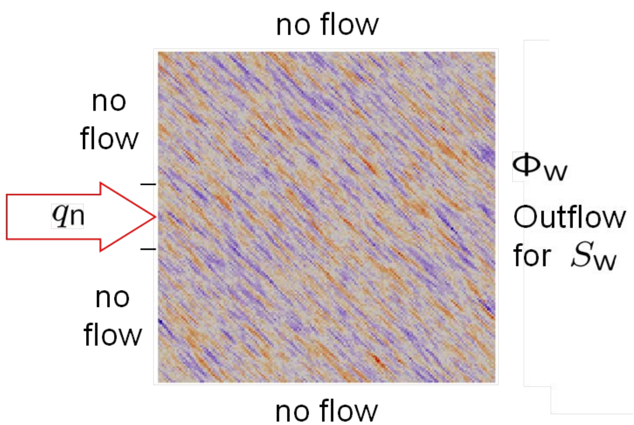


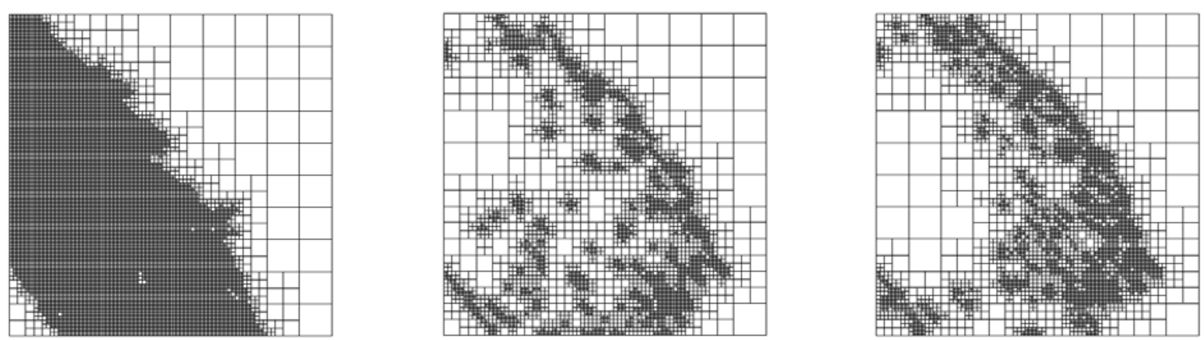
fine-scale reference

adaptive, multi-scale, conservative refinement

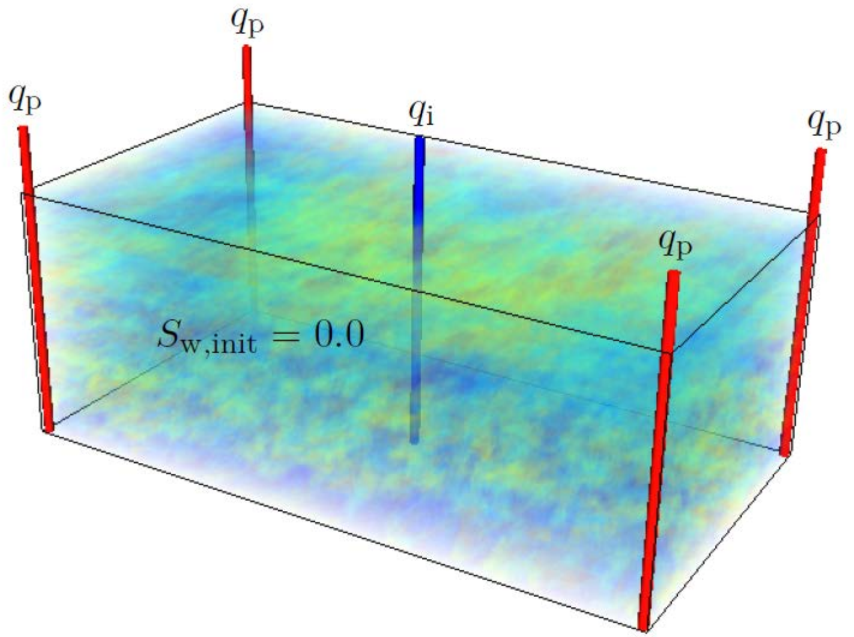
adaptive, multi-scale, intermediate refinement

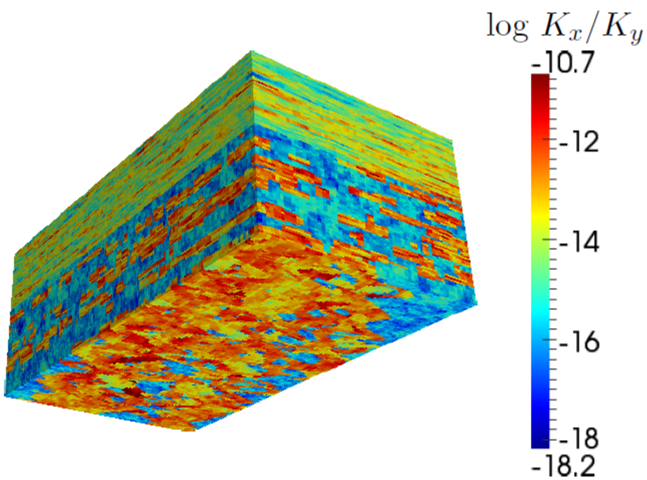
adaptive, multi-scale, lax refinement

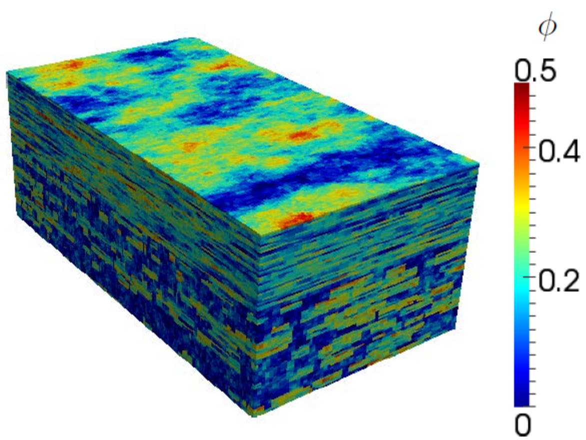


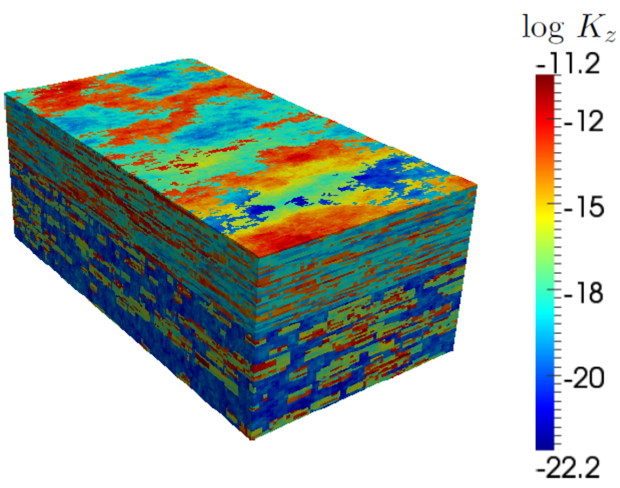


SPE 10 Model 2 (Christie and Blunt, 2001)



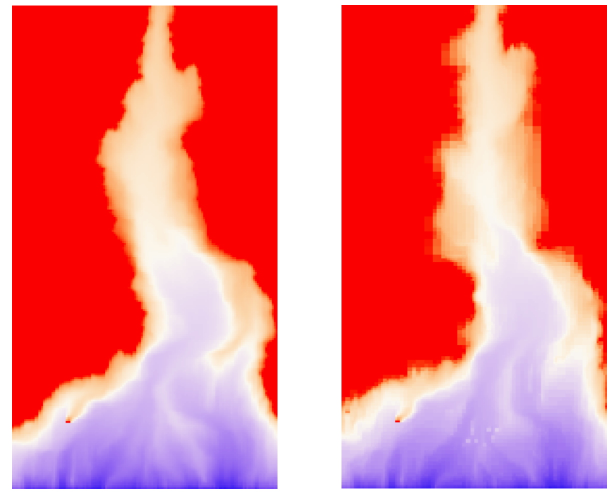




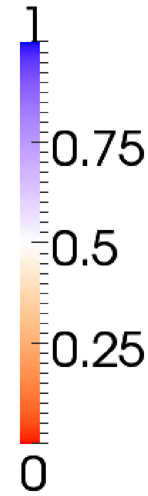


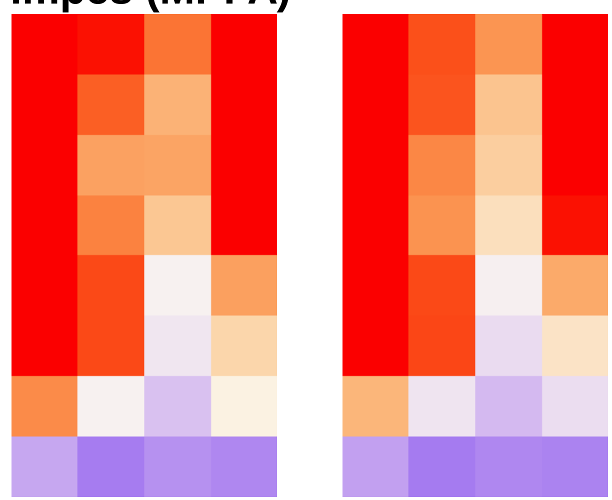
Combination of all indicators

- One indicator which accounts for errors in the saturation transport (local sat gradient)
- One indicator which accounts for errors in the flow field (total velocity)
- Multi-scale: Permeability upscaling + adaptive grid



Fine scale impes (MPFA) **Multi-scale**

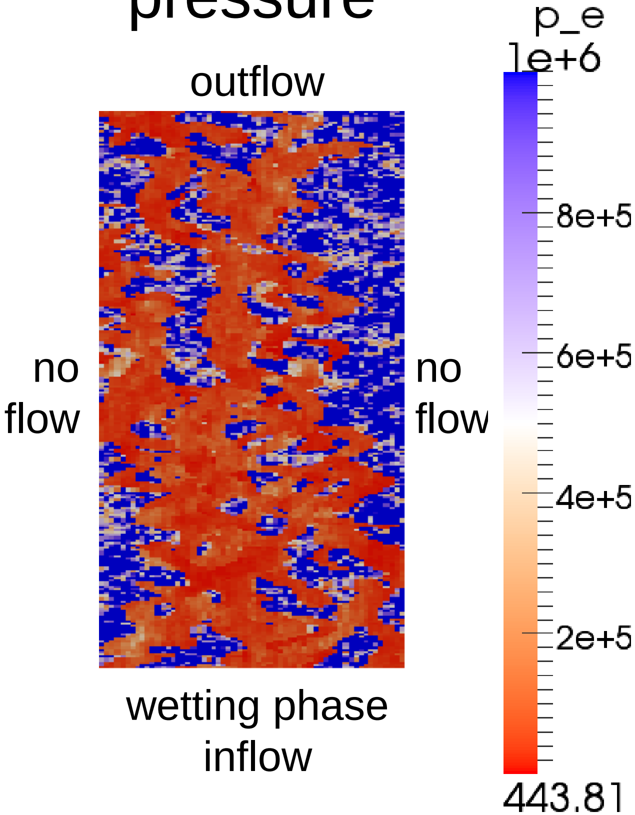


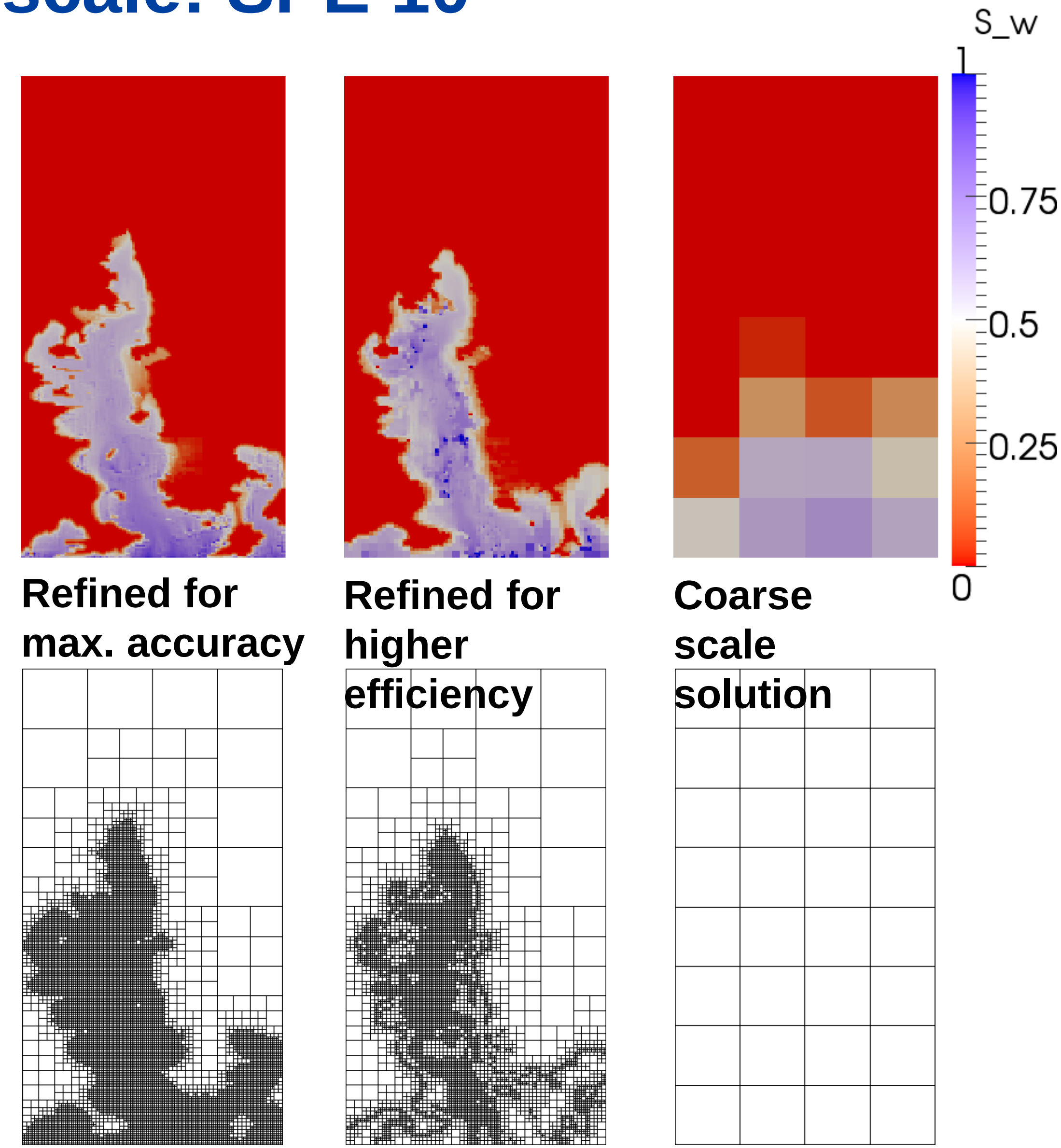


Solutions averaged to the coarse scale grid

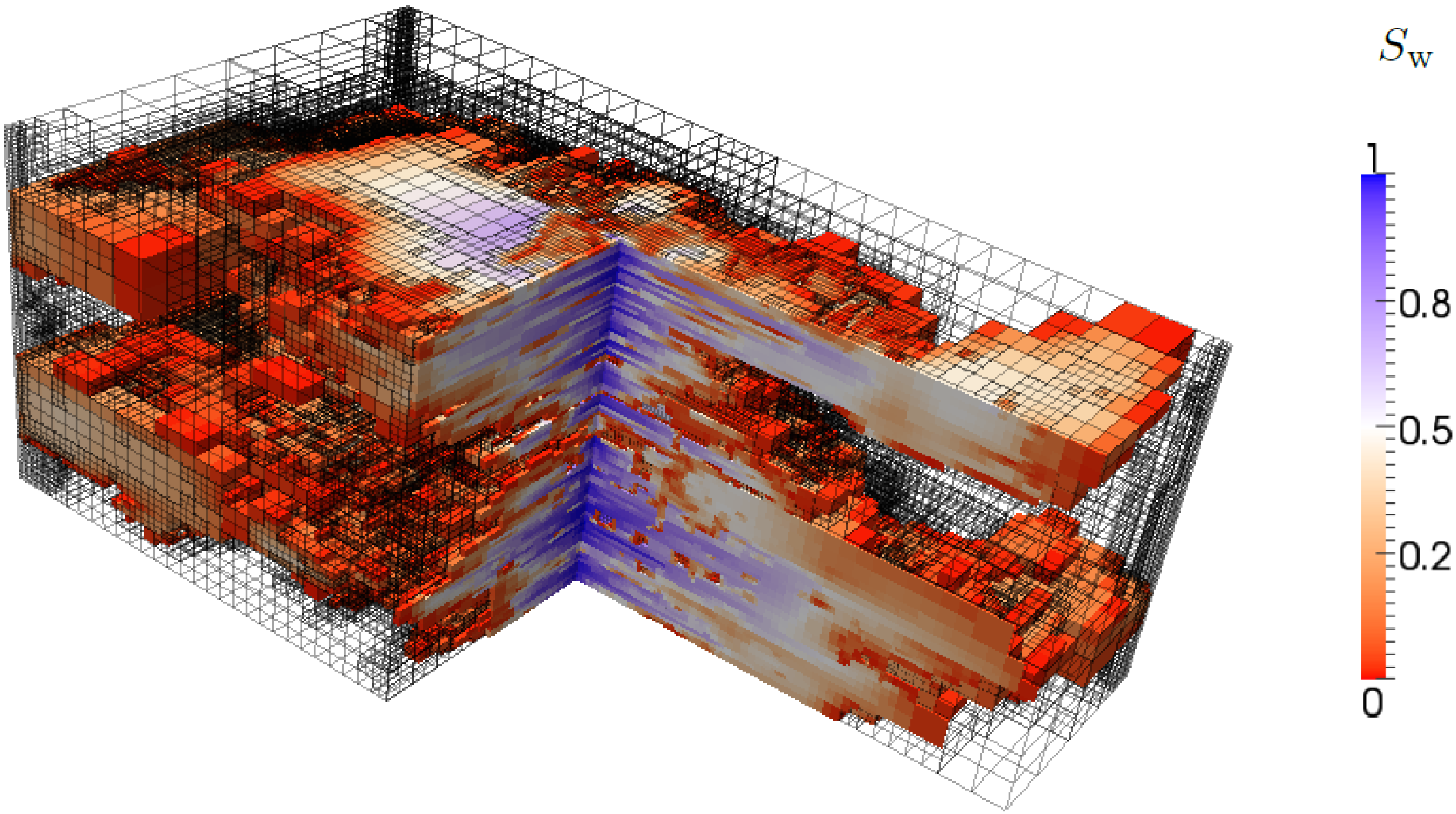
Multi-scale: SPE 10

- Layer from bottom formation
- With capillary pressure

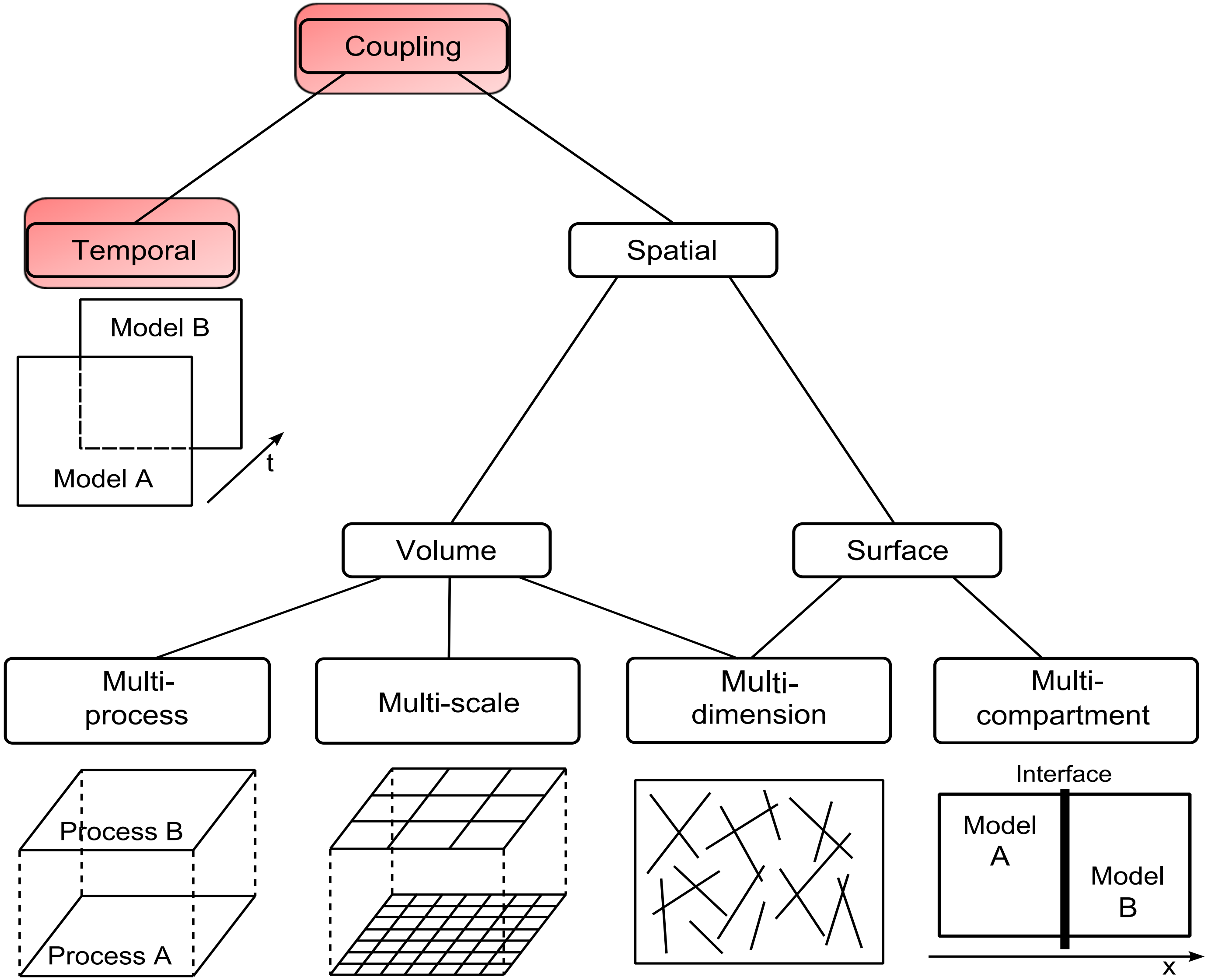




SPE 10 Model 2



Adaptive time discretization



Transport equations

Two-Phase Flow equations:

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot v_w = q_w$$

$$\phi \frac{\partial S_n}{\partial t} + \nabla \cdot v_n = q_n$$

Fractional Flow Formulation:

$$\nabla \cdot [-\lambda_t \mathbf{K}(\nabla \Phi_w + f_n \nabla \Phi_c)] = q_w + q_n$$

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot v_w = q_w$$

Phase and Capillary Potential:

$$\Phi_\alpha = p_\alpha + \rho_\alpha g z, \; \alpha = w, n, \; \Phi_c = p_c + (\rho_n - \rho_w) g z$$

elliptic + parabolic equation

↓

Fully Implicit Methods

↓

Sequential Methods

Discretization of Fractional Flow Equation

IMPES

$$\mathbf{A}_\Phi(\mathbf{S}_w^n)\mathbf{\Phi}_w^{n+1} + \mathbf{A}_c(\mathbf{S}_w^n)\mathbf{\Phi}_c(\mathbf{S}_w^n) = \mathbf{Q}_\Phi^{n+1}$$

$$\mathbf{M}\frac{\mathbf{S}_w^{n+1} - \mathbf{S}_w^n}{\Delta t^n} + \mathbf{A}_w(\mathbf{S}_w^n)\mathbf{\Phi}_w^{n+1} = \mathbf{Q}_w^{n+1}$$

Assumptions: equations are weakly coupled

Problems: CFL restrictions

FI

$$\mathbf{A}_\Phi(\mathbf{S}_w^{n+1})\mathbf{\Phi}_w^{n+1} + \mathbf{A}_c(\mathbf{S}_w^{n+1})\mathbf{\Phi}_c(\mathbf{S}_w^{n+1}) = \mathbf{Q}_\Phi^{n+1}$$

$$\mathbf{M}\frac{\mathbf{S}_w^{n+1} - \mathbf{S}_w^n}{\Delta t^n} + \mathbf{A}_w(\mathbf{S}_w^{n+1})\mathbf{\Phi}_w^{n+1} = \mathbf{Q}_w^{n+1}$$

Assumptions: no

Problems: convergence of solver

IMPSAT

$$\mathbf{A}_\Phi(\mathbf{S}_w^n)\mathbf{\Phi}_w^{n+1} + \mathbf{A}_c(\mathbf{S}_w^n)\mathbf{\Phi}_c(\mathbf{S}_w^n) = \mathbf{Q}_\Phi^{n+1}$$

$$\mathbf{M}\frac{\mathbf{S}_w^{n+1} - \mathbf{S}_w^n}{\Delta t^n} + \mathbf{A}_w(\mathbf{S}_w^{n+1})\mathbf{\Phi}_w^{n+1} = \mathbf{Q}_w^{n+1}$$

Assumptions: weakly coupled

Problems: time step restrictions

IMPCAP

$$\mathbf{A}_\Phi(\mathbf{S}_w^n)\mathbf{\Phi}_w^{n+1} + \mathbf{A}_c(\mathbf{S}_w^n)\mathbf{\Phi}_c^{\text{approx}}(\mathbf{S}_w^{n+1}) = \mathbf{Q}_\Phi^{n+1}$$

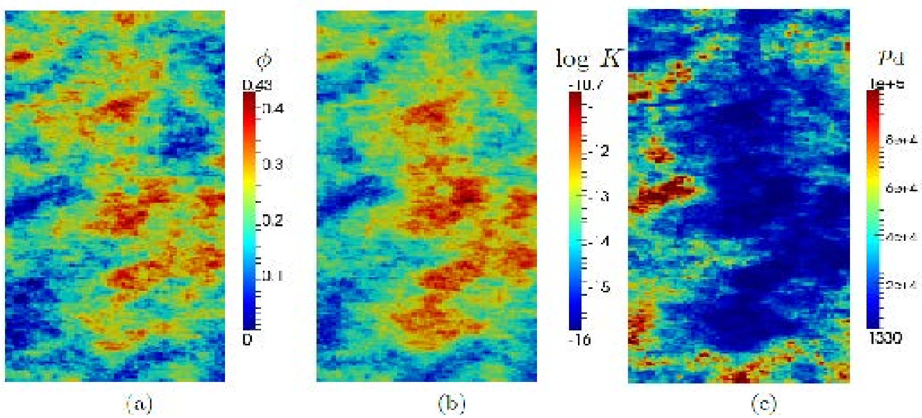
$$\mathbf{M}\frac{\mathbf{S}_w^{n+1} - \mathbf{S}_w^n}{\Delta t^n} + \mathbf{A}_w(\mathbf{S}_w^n)\mathbf{\Phi}_w^{n+1} = \mathbf{Q}_w^{n+1}$$

Assumptions: weakly coupled

Problems: CFL, expensive assembling

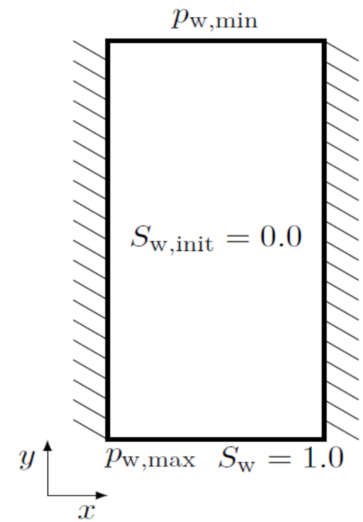
Comparison of Efficiency and Accuracy

SPE 10, Layer 15



(a) porosity (b) permeability (c) entry pressure

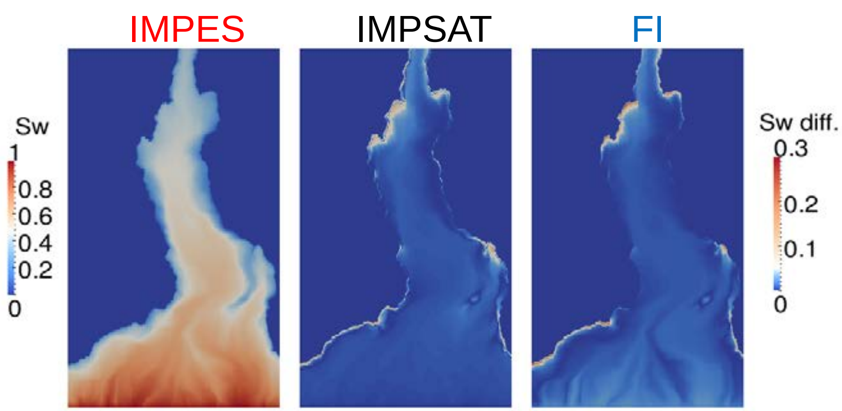
Problem Setting



$p_{w,min} = 20$ MPa

$p_{w,max} = 40$ MPa

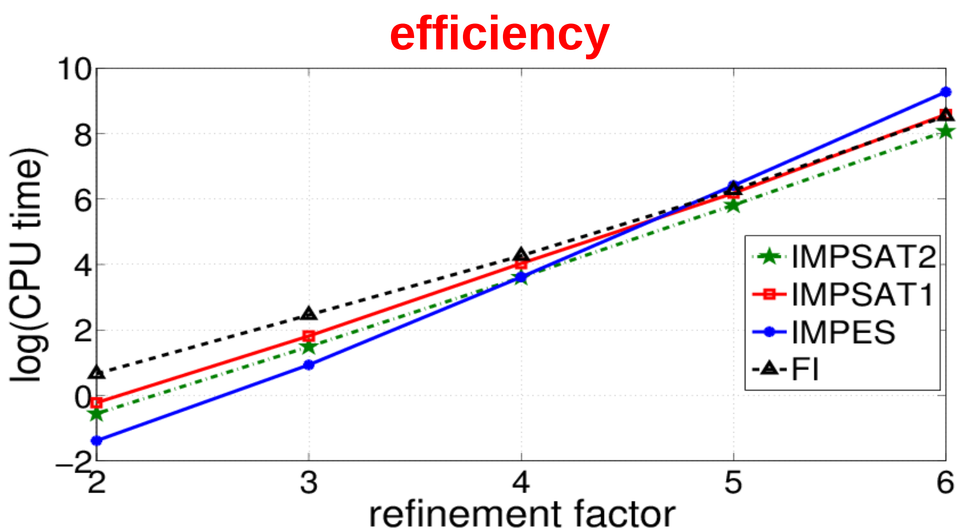
Solution without capillary pressure:



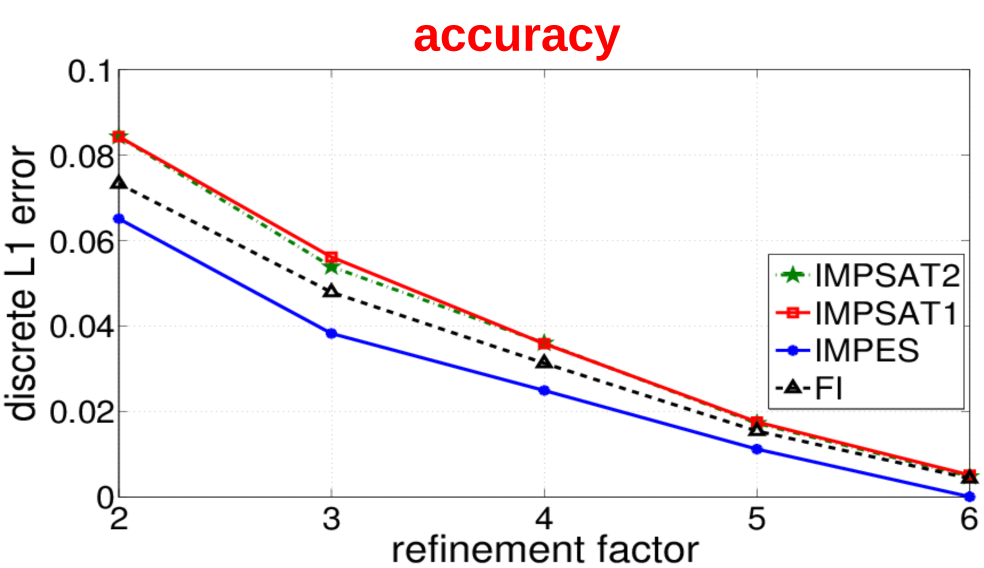
- similar results
- FI and IMPSAT: more numerical diffusion

Comparison of Efficiency and Accuracy

- IMPES fastest up to ~ 10^4 cells, lowest order
- IMPSAT faster than FI
- FI highest efficiency order

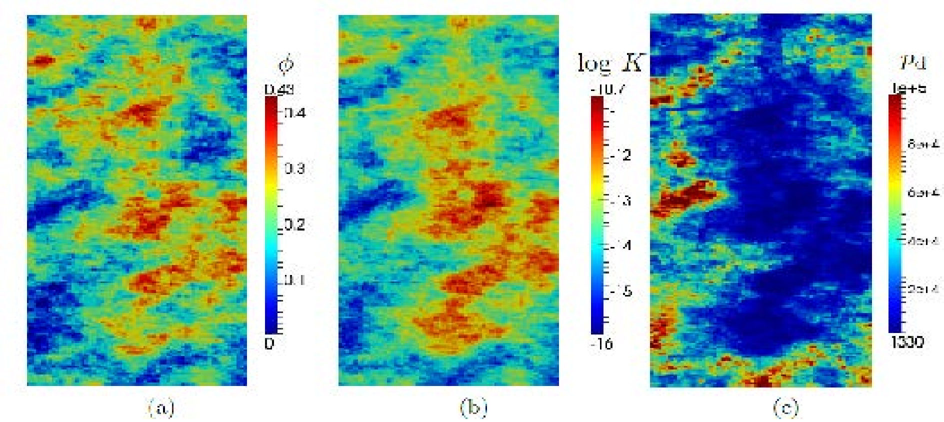


- All methods converge
- FI higher accuracy than IMPSAT
- Jocobian reassembling has no effect on accuracy



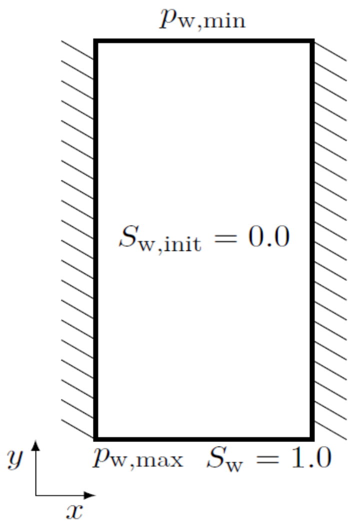
Problem with capillary pressure

SPE 10, Layer 15



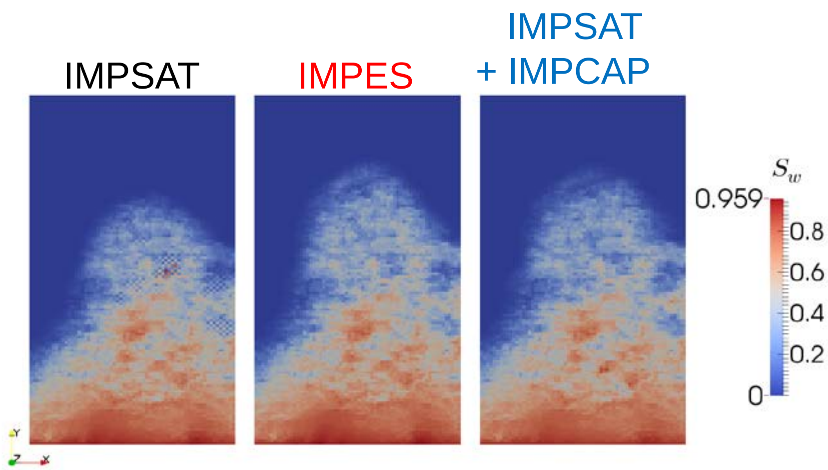
(a) porosity (b) permeability (c) entry pressure

Problem Setting

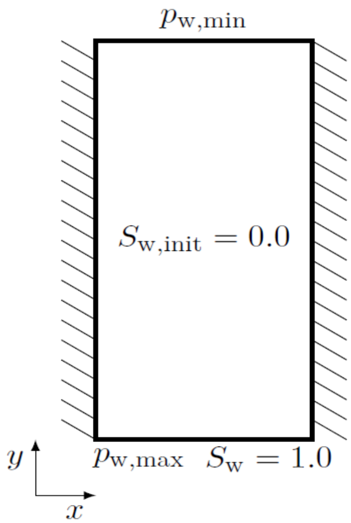


$$p_{w,min} = 25 \text{ MPa}$$
$$p_{w,max} = 26 \text{ MPa}$$

Problem with capillary pressure



Problem Setting



$$p_{w,min} = 25 \text{ MPa}$$
$$p_{w,max} = 26 \text{ MPa}$$

IMPSAT:
- Wrong front movement
- Assumption of weakly coupled equations is not fullfilled

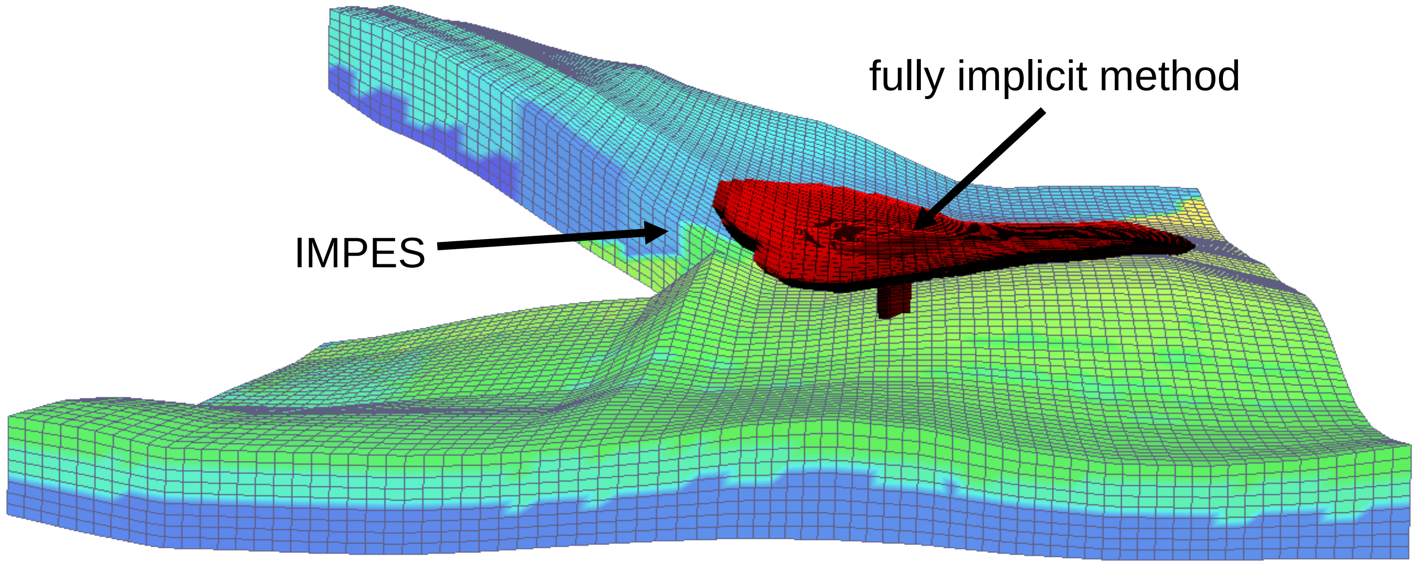
IMPES:
- CFL Coats criteria produces small time step sizes
- Equations are only for small steps weakly coupled

IMPSAT + IMPCAP:
- Correct solution
- Bigger time step sizes are possible
- Loss of sparsity pattern for matrices

Motivation Adaptive Implicit Method (AIM)

For each numerical method you can design a poblem where it performs bad

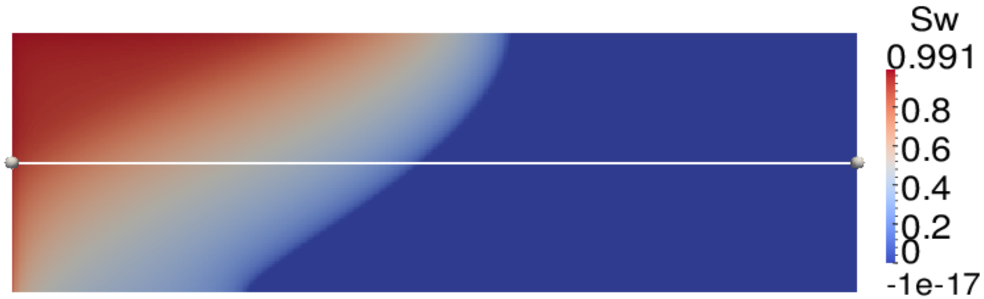
→ **adapt numerical methods**



Regions are changing per time, they move with the physical processes

Two-phase flow with gravity and capillary pressure

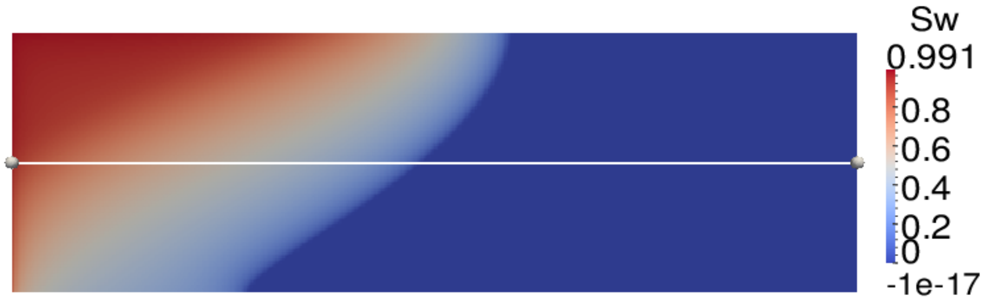
S_w



x

More complex example

including gravity + capillary pressure



	T_{CPU}	$\Delta t_{\text{average}}$	time steps
IMPES	$\approx$ 88h	0.035h	112928
AIM	$\approx$ 0.6h	32h	125

no optimal AIM example

Next Steps / Outlook

- Implementation of MPFA for AIM
- Further investigation of AIM approach
- Increase convergence of solver / better preconditioner:

$$\mathbf{R_1} = \mathbf{A}_\Phi(\mathbf{S}_w^{n+1})\mathbf{\Phi}_w^{n+1} + \mathbf{A}_c(\mathbf{S}_w^{n+1})\mathbf{\Phi}_c(\mathbf{S}_w^{n+1}) - \mathbf{Q}_\Phi^{n+1}$$ elliptic

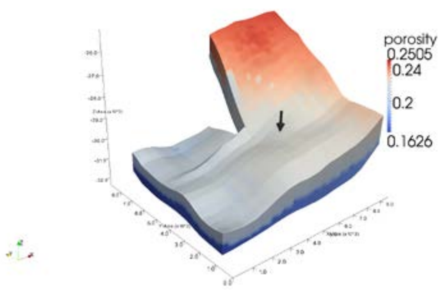
$$\mathbf{R_2} = \mathbf{M}\frac{\mathbf{S}_w^{n+1} - \mathbf{S}_w^n}{\Delta t^n} + \mathbf{A}_w(\mathbf{S}_w^{n+1})\mathbf{\Phi}_w^{n+1} - \mathbf{Q}_w^{n+1}$$ parabolic

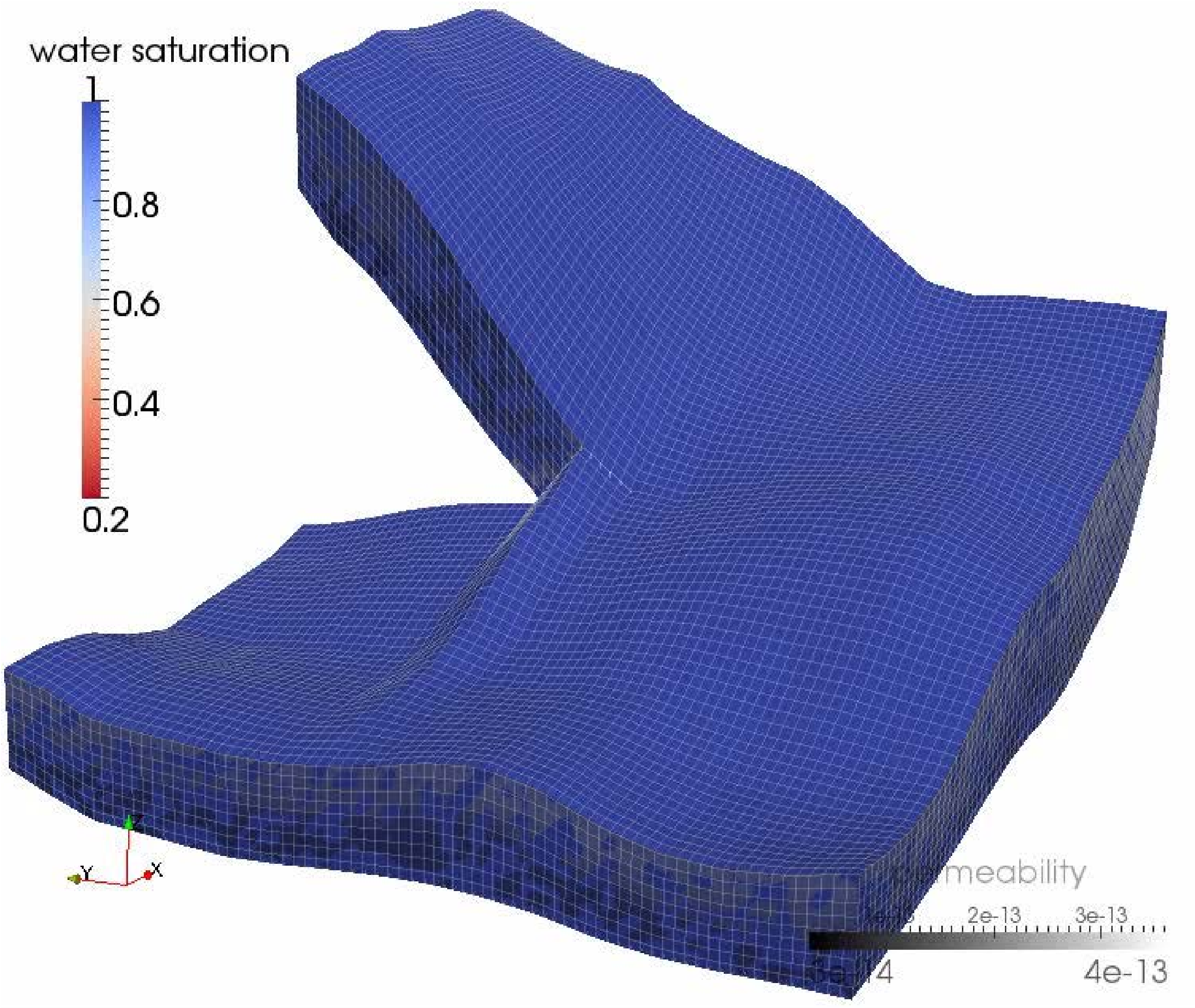
$$\mathbf{X} = (\mathbf{P_w}, \mathbf{S_w}) \qquad \mathbf{J_R} = \frac{\partial \mathbf{R}}{\partial \mathbf{X}} = \begin{pmatrix} \mathbf{A_{1,P_w}} & \mathbf{A_{1,S_w}} \\ \mathbf{A_{2,P_w}} & \mathbf{A_{2,S_w}} \end{pmatrix}$$

→ **use different preconditioner for elliptic and parabolic equation**

→ **CPR-AMG preconditioner**

Johansen Formation





Summary – State of current work

- **Adaptive grid refinement**: Hanging nodes, MPFA L-method (3D)
- **Numerical upscaling** of phase permeabilities: tensorial relative permeabilities
- **Treatment** of **tensorial relative permeabilities** using MPFA
- **Multi-scale**: Combination of numerical upscaling and adaptive grid refinement

Future work:

- **Adaptive time discretization** (IMPES, sequential and fully implicit)
- **Large scale simulation**
- Include **multi-physics** concept (e.g. 3-phase – 2-phase flow)

The End

Universität Stuttgart

SimTech
Cluster of Excellence

International Research Training Group
nupus

References

Faigle B., et al. (2013): *Efficient multi-physics modelling with adaptive grid-refinement using a MPFA method,* Computational Geosciences, 2014

Wolff M., et al., *An adaptive multi-scale approach for modeling two-phase flow in porous media including capillary pressure*, Water Resour Res, 2013

Wolff M., et al., *Multi-point flux approximation L-method in 3D: numerical convergence and application to two-phase flow through porous media*, De Gruyter, 2013

Helmig, R., A. Ebigbo, M. Wolff, B. Flemisch, H. Class: Model coupling for multiphase flow in porous media. Advances in Water Resources, 2012.

Wolff M., et al., *Treatment of tensorial relative permeabilities with multipoint flux approximation*, Int J Numer Anal Mod, 2012

dumux.org

http://dune-project.org/