Modeling and Simulation in Porous Media

in Honor of Jean Roberts & Jérôme Jaffré

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Total Differential interpolation of Two-phase Data for Three-phase Compressible Flows

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Summary

- Three-phase compressible equations
- Choice of pressure unknown :
  water, oil or gas versus global ?
- TD-condition $\Leftrightarrow$ existence of a global capillary function P_{cg}
- Interpolation of TD-three-phase data from two-phase data
- Numerical results
- Conclusions

Three-phase Compressible equations

- Classical resolution (1=water, 2=oil, 3=gas) :

solve for the oil pressure P_2 the “pressure equation” :

$$\frac{\partial}{\partial t}\{\phi \sum_{j=1}^{3} B_j\, S_j\} + \nabla \cdot q = 0 \ ,$$

where q is the global volumetric flow vector:

$$q = \sum_{j=1}^{3} \varphi_j = -Kd\{\nabla P_2 + f_1 \nabla P_c^{12} + f_3 \nabla P_c^{32} - \rho g \nabla Z\}$$

$$\begin{cases} d(s_1, s_3, p_2) & = & \sum_{j=1}^{3} kr_j d_j & = global\ mobility, \\ f_j(s_1, s_3, p_2) & = & kr_j d_j / \lambda & = j^{th}\ fractional\ flow\ ,\ \sum_{j=1}^{3} f_j = 1, \\ \rho(s_1, s_3, p_2) & = & \sum_{j=1}^{3} f_j \rho_j & = global\ density. \end{cases}$$

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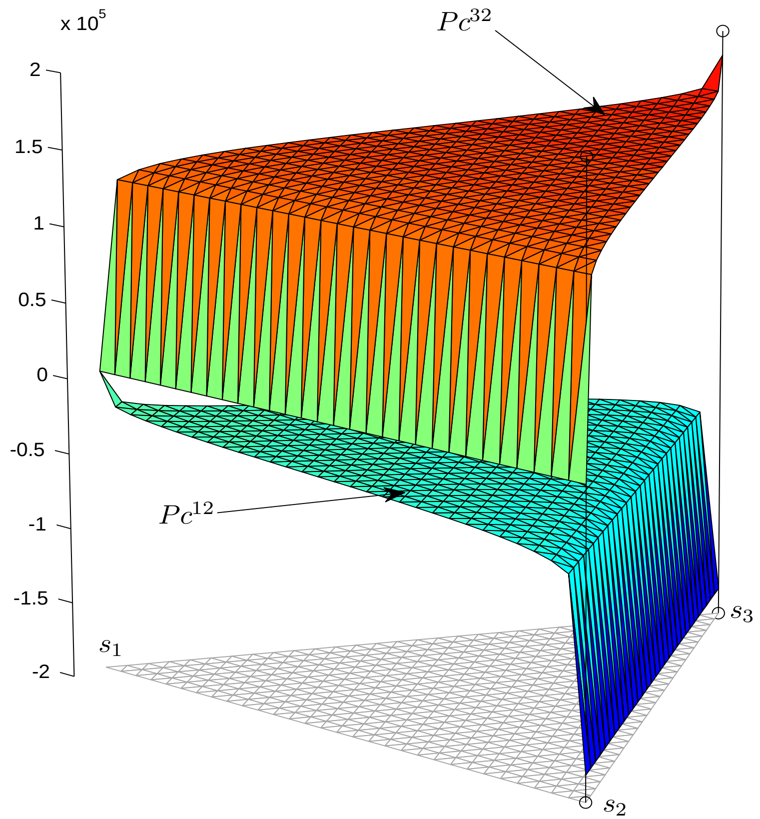
Is oil pressure a better unknown than water or gas pressure ?

Pressure unknown: water, oil or gas? (1)

- Capillary pressures (Van Genuchten):

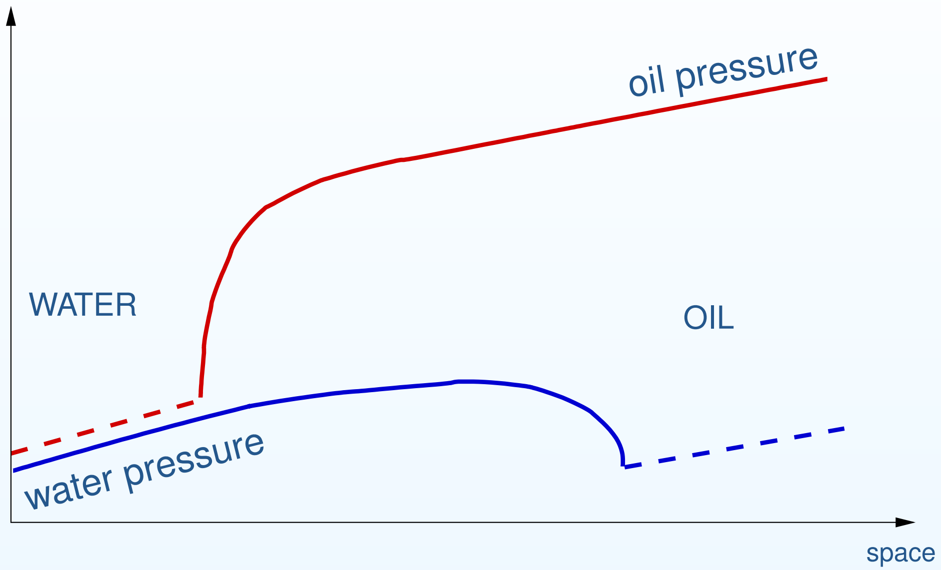
they depend only on the wetting phase saturation !



top: $P_c^{32} = P_{gas} - P_{oil}$

bottom : $P_c^{12} = P_{water} - P_{oil}$

Let us have a dream...



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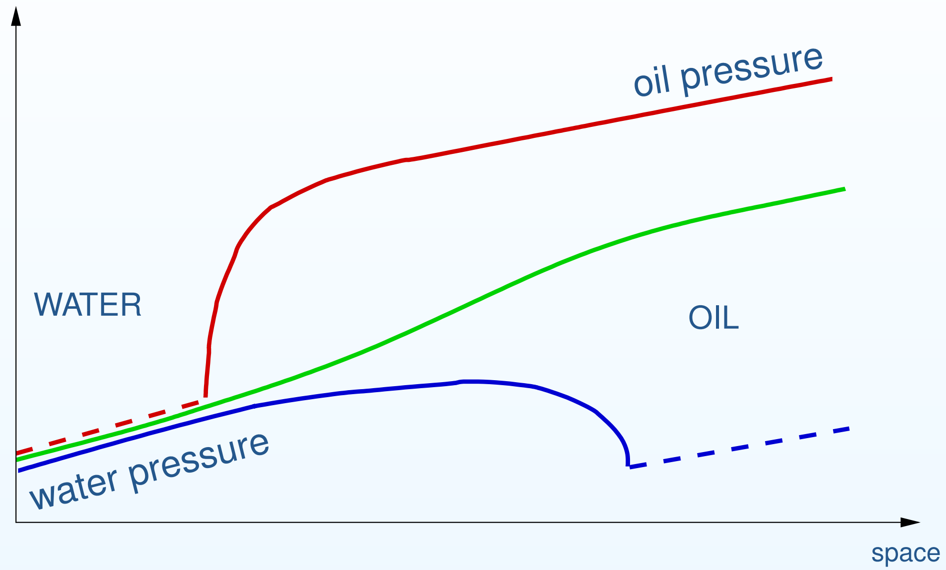

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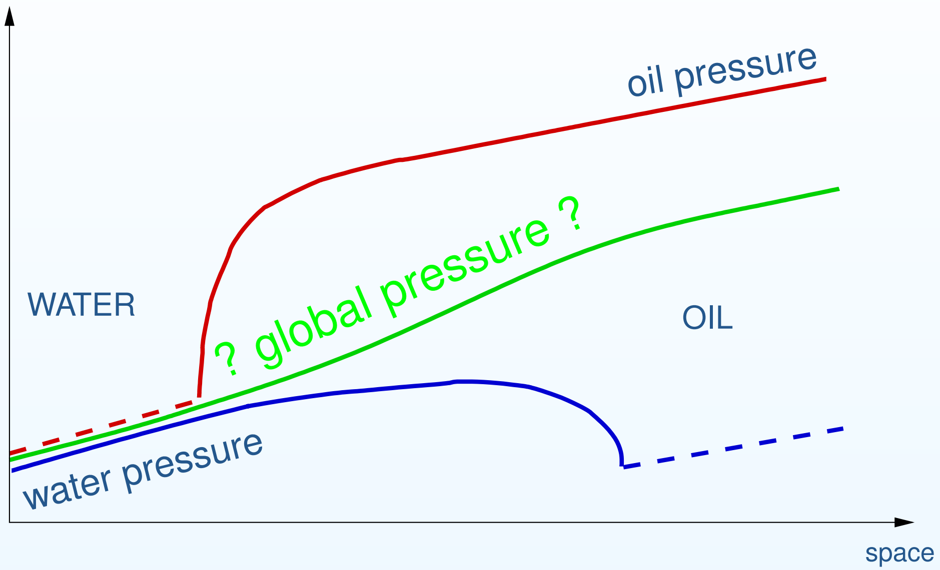

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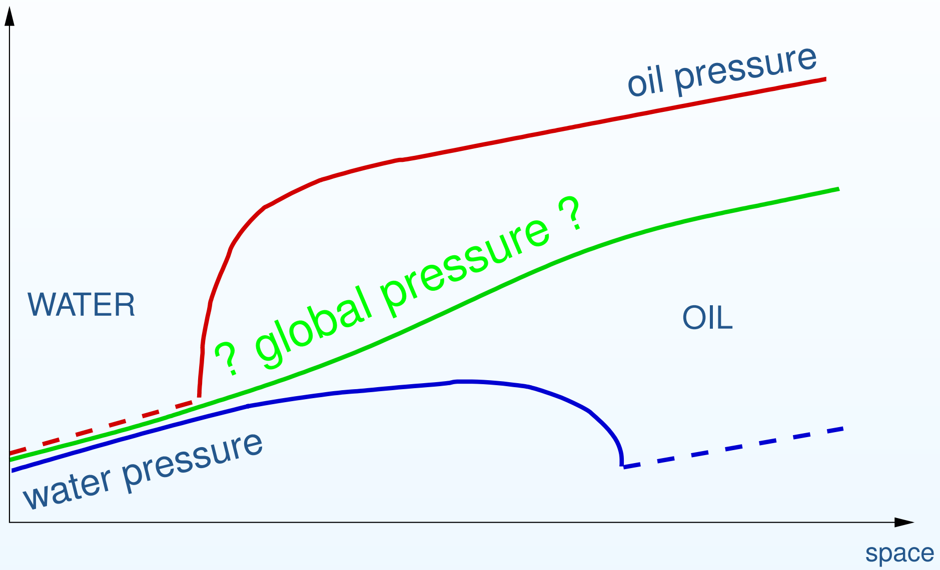

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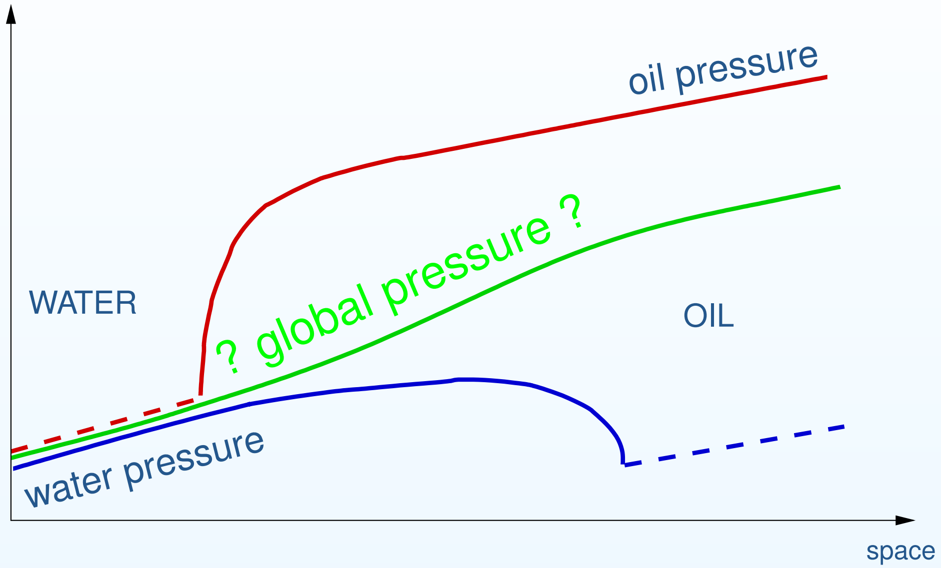

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- There exists a Global Capillary Pressure function P_{cg} s. t. :

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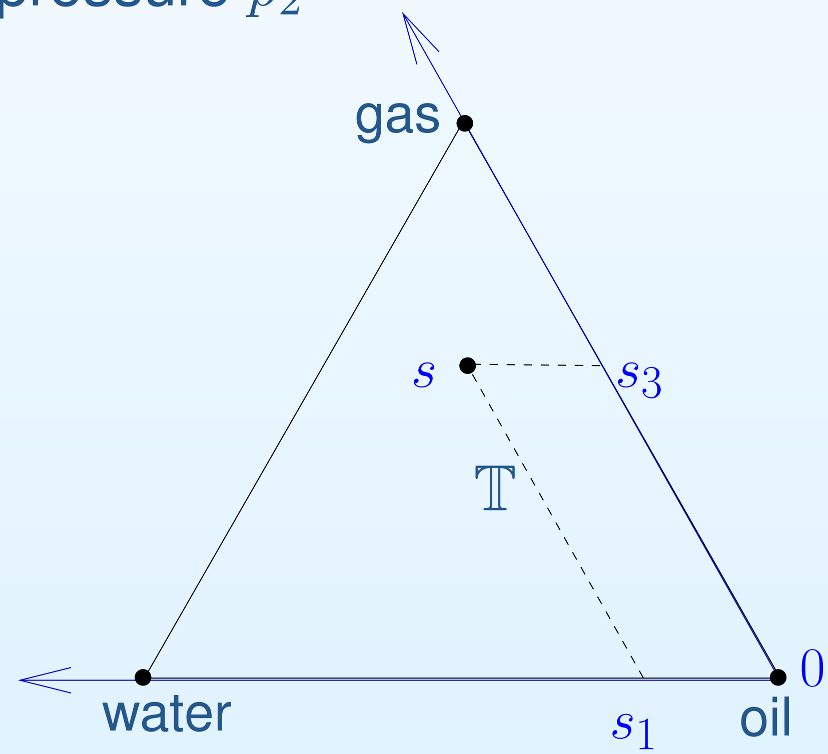

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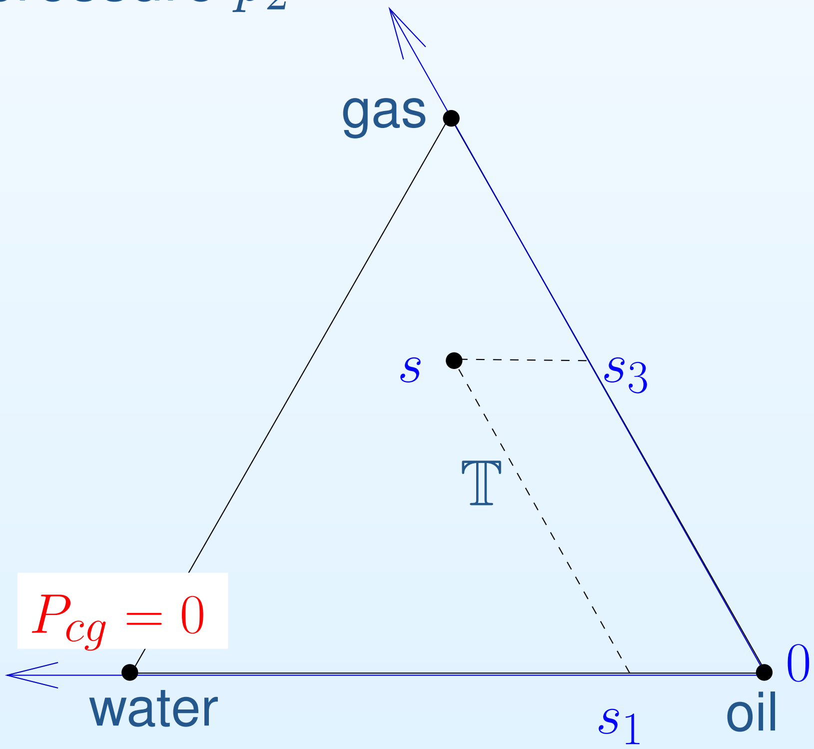

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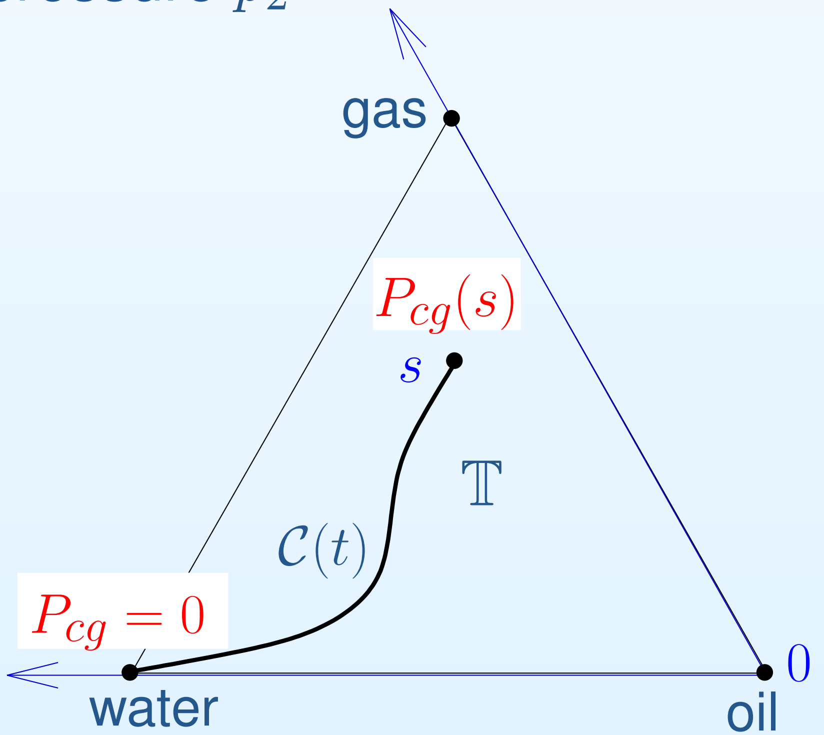

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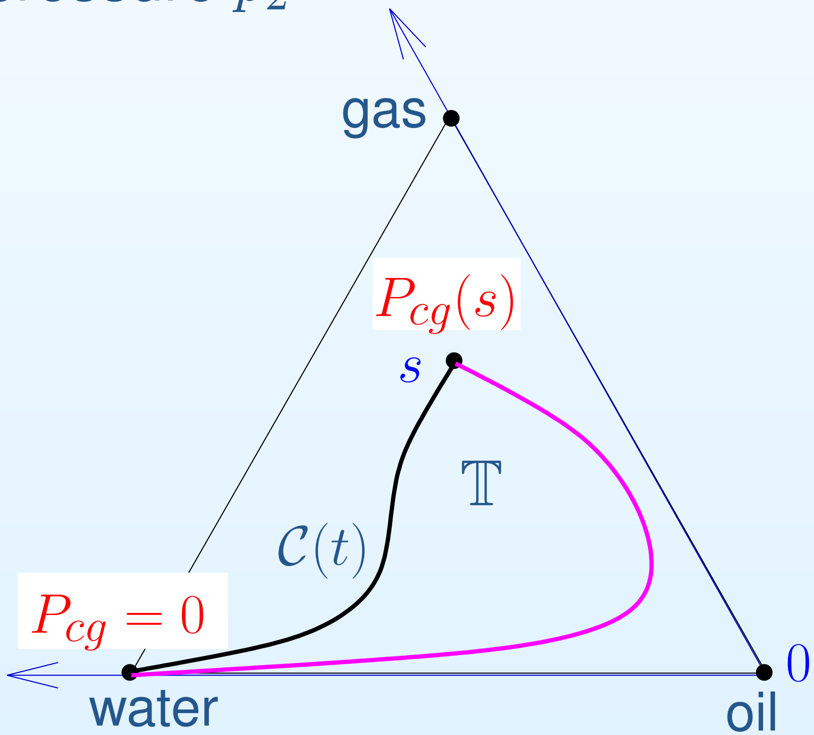

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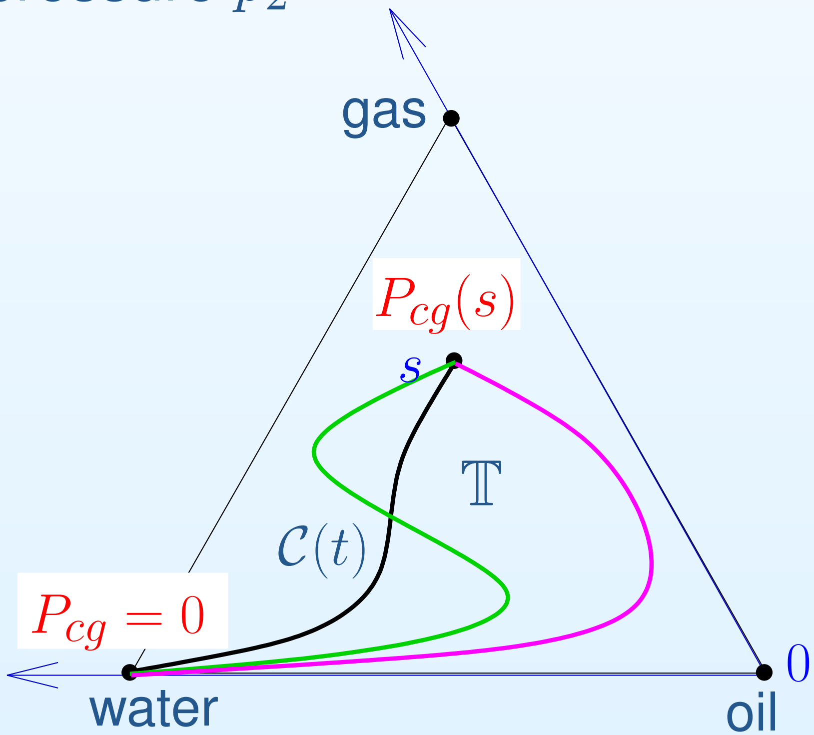

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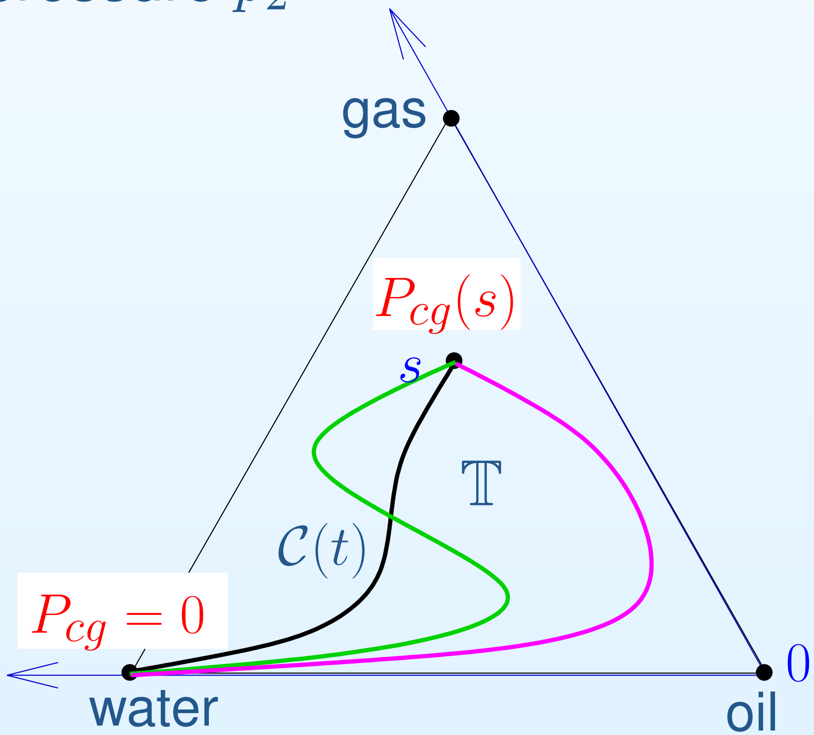
$$\Downarrow$$

TD-condition on $kr_1, kr_2, kr_3, P_c^{12}, P_c^{32}$



Global pressure formulation :

- When P_{cg} exists, the Global Pressure P is defined by :

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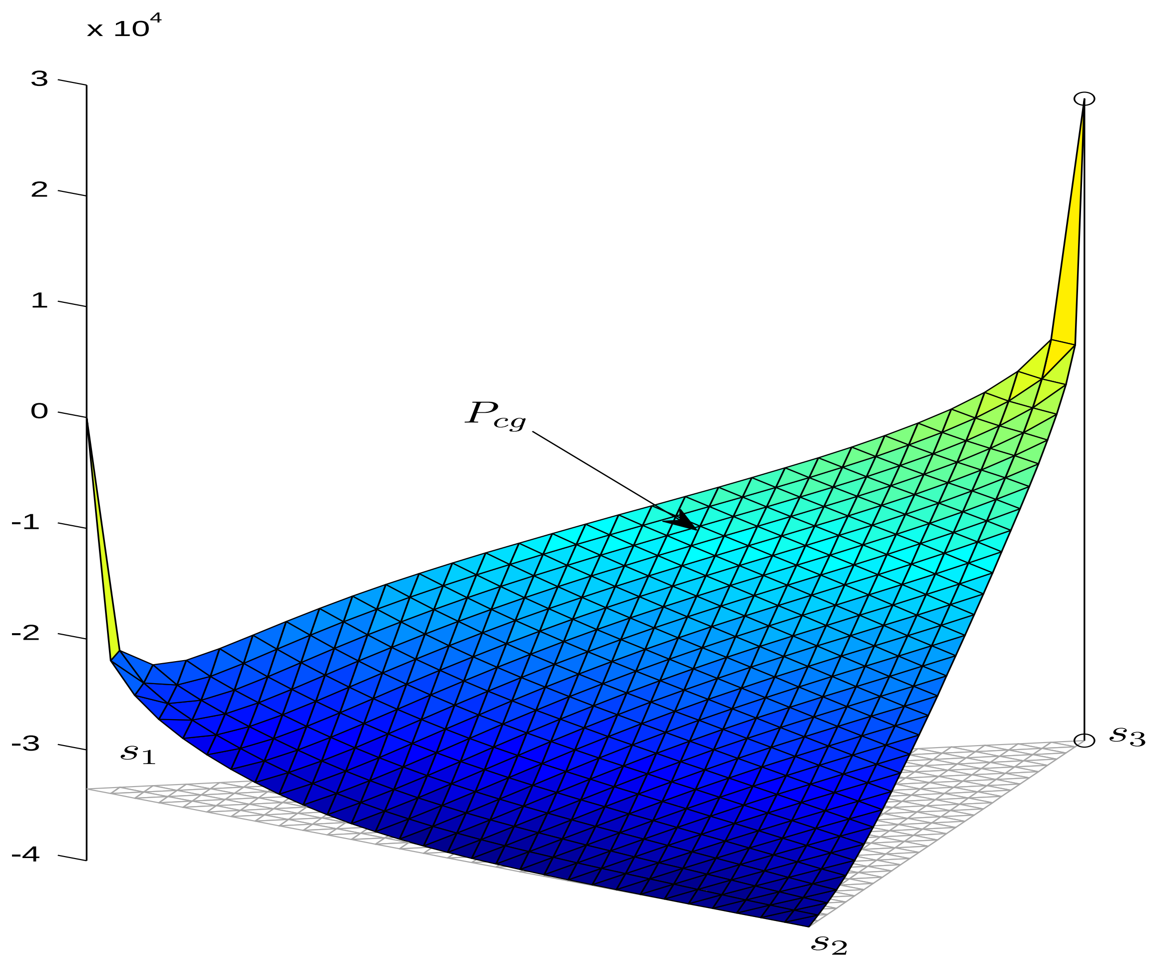
- For a single compressible fluid (gas), one has: :

$$0 < \partial P_{cg}/\partial P \leq 1 \quad \text{over } \mathbb{T}$$

- Numerical results show that:

$$0 < \partial P_{cg}/\partial P \leq 10^{-4} \qquad \text{hence :} \qquad 1 - \partial P_{cg}/\partial P \simeq 1$$

Global Capillary Pressure function $P_{cg} = P - P_2$



Degrees of freedom for TD relative permeabilities :

At each global pressure level p

- one can choose freely only two functions over $\mathbb{T}$:

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$$\begin{cases} kr_j(s,p) & = & f_j(s,p)d(s,p)/d_j\big(p - P_{cg}(s,p) + P_c^{j2}(s_j)\big) \quad j = 1,3 \; , \\ kr_2(s,p) & = & (1 - f_1(s,p) - f_3(s,p))d(s,p)/d_2\big(p - P_{cg}(s,p)\big) \; , \end{cases}$$

Construction of TD-relative permeabilities

Available experimental data :

- for each fluid $j = 1, 2, 3$:
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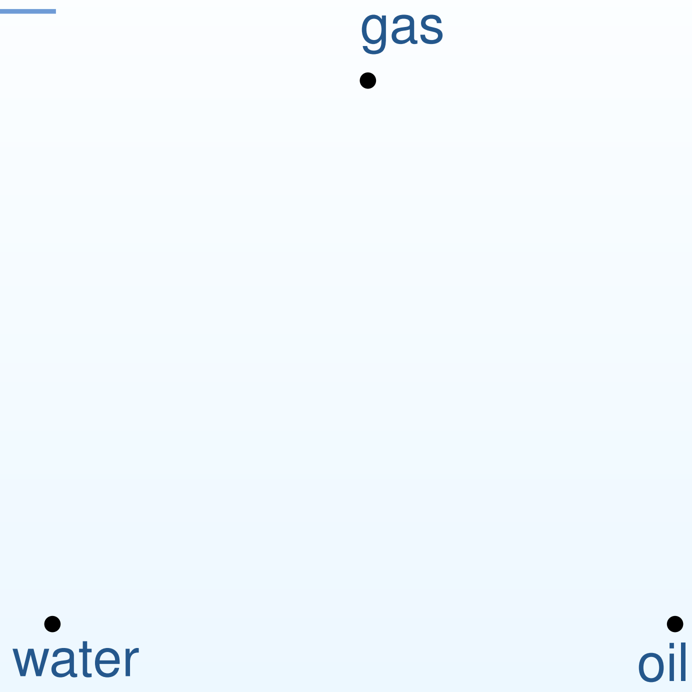

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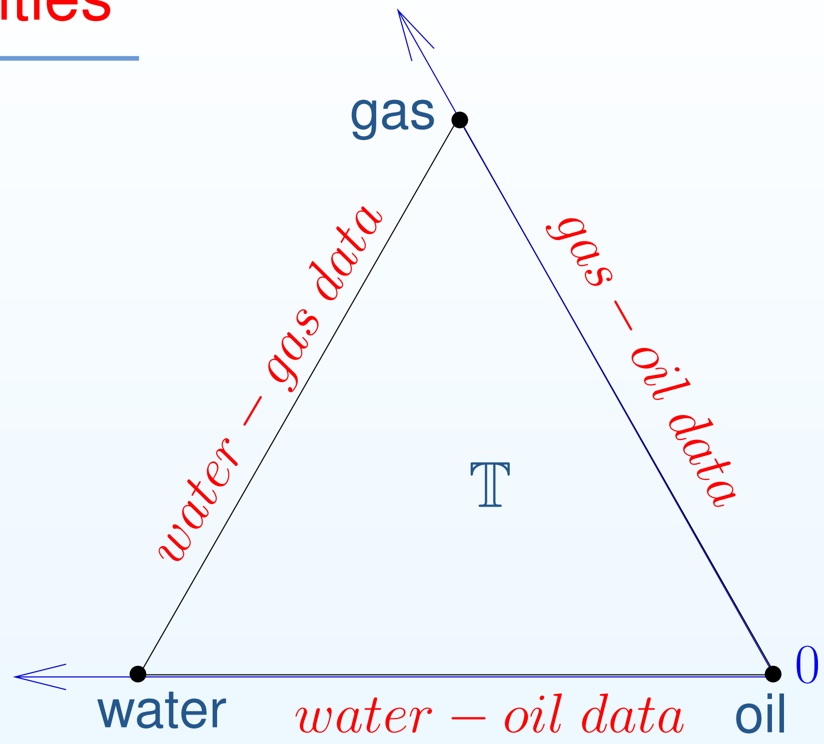

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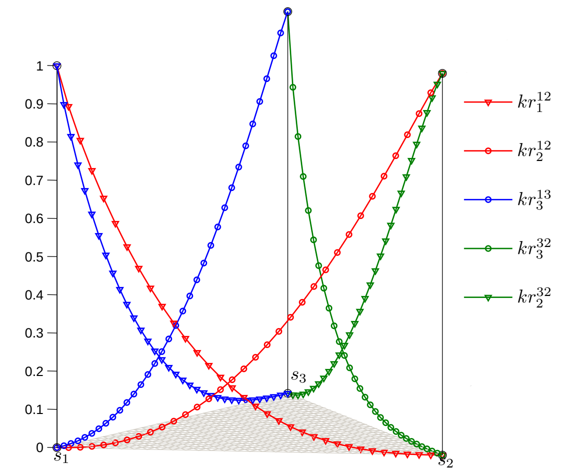

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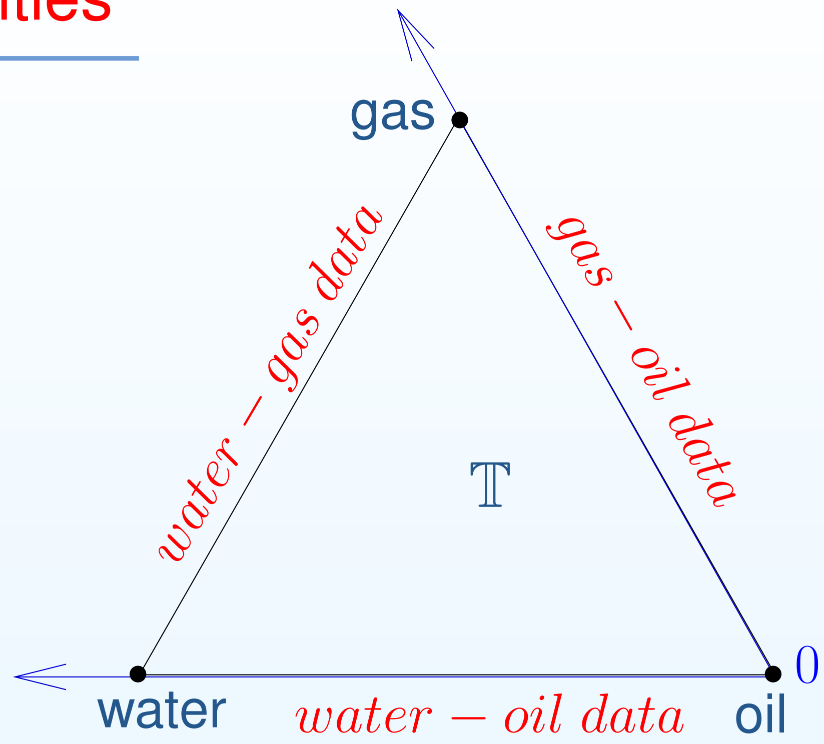

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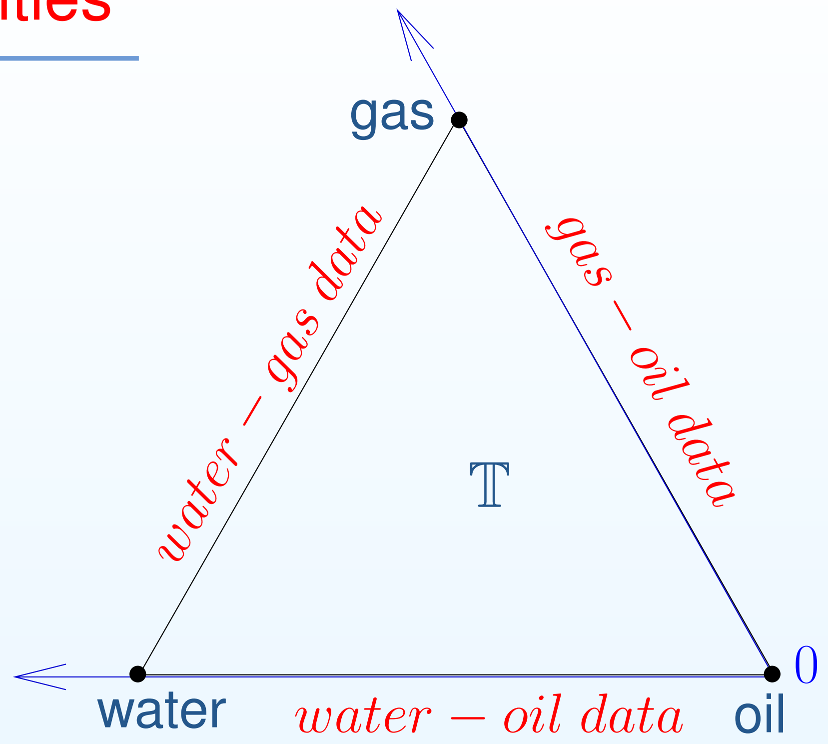

Question :

can TD-three-phase relative permeabilities on $\mathbb{T}$

honor two-phase data on $\partial\mathbb{T}$?

Honoring two-phase data : 1 Determine P_{cg} on $\partial\mathbb{T}$

- Let p be a given global pressure level.
- Recall : on any curve $\mathcal{C} : [0,1] \leadsto \mathbb{T}$, $P_{cg}(\mathcal{C}(t))$ satisfies :

$$(1)\ \frac{dP_{cg}}{dt} = f_1(\mathcal{C}, \underbrace{p - P_{cg}}_{P_2}) \frac{dP_c^{12}}{ds_1}(\mathcal{C}_1)\, \mathcal{C}_1' + f_3(\mathcal{C}, \underbrace{p - P_{cg}}_{P_2}) \frac{dP_c^{32}}{ds_3}(\mathcal{C}_3)\, \mathcal{C}_3' \ ,$$

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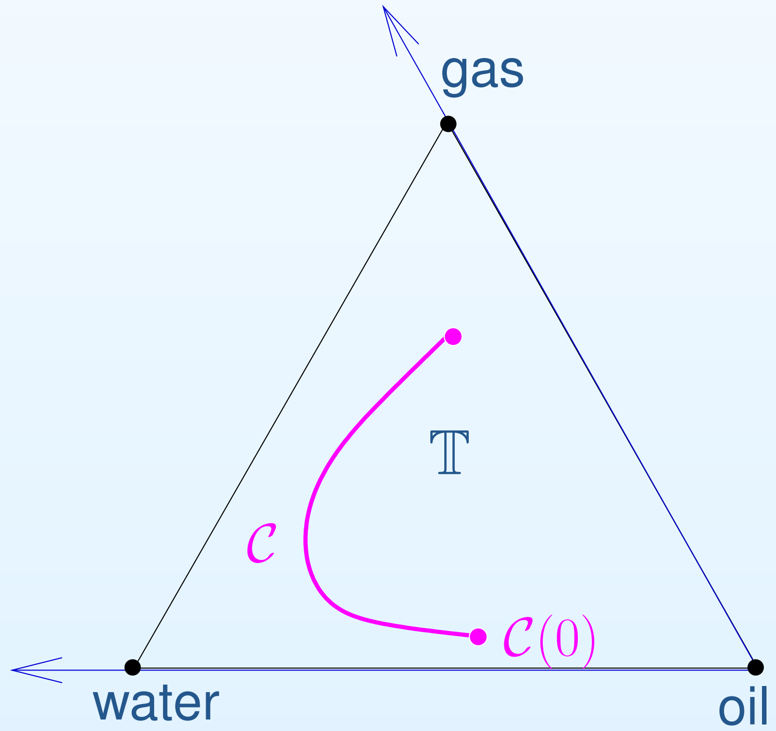

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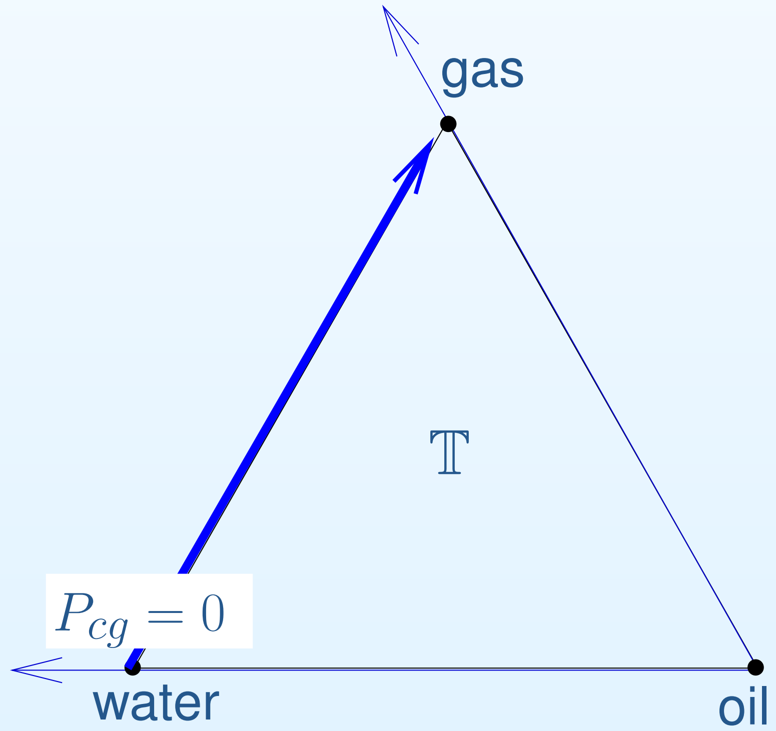

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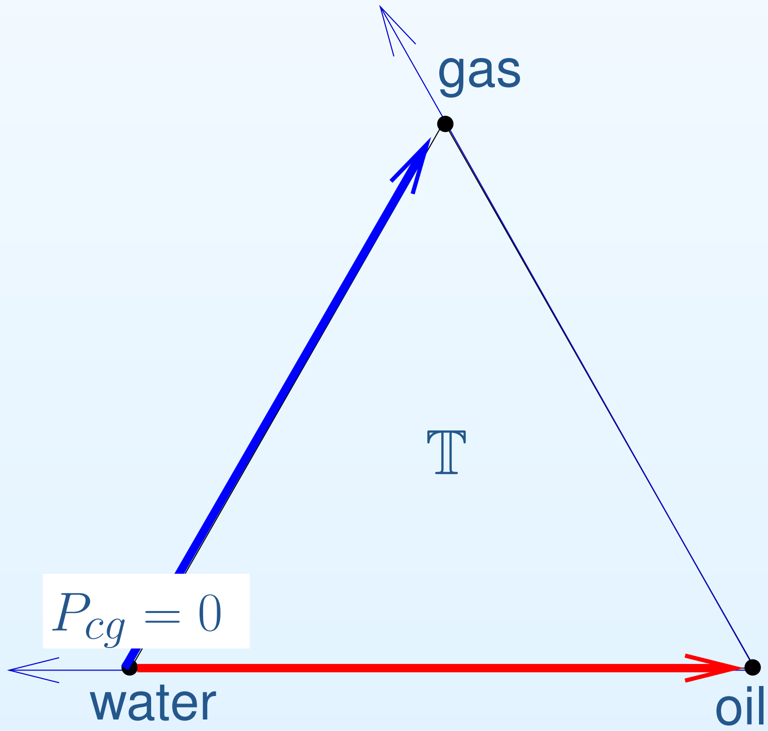

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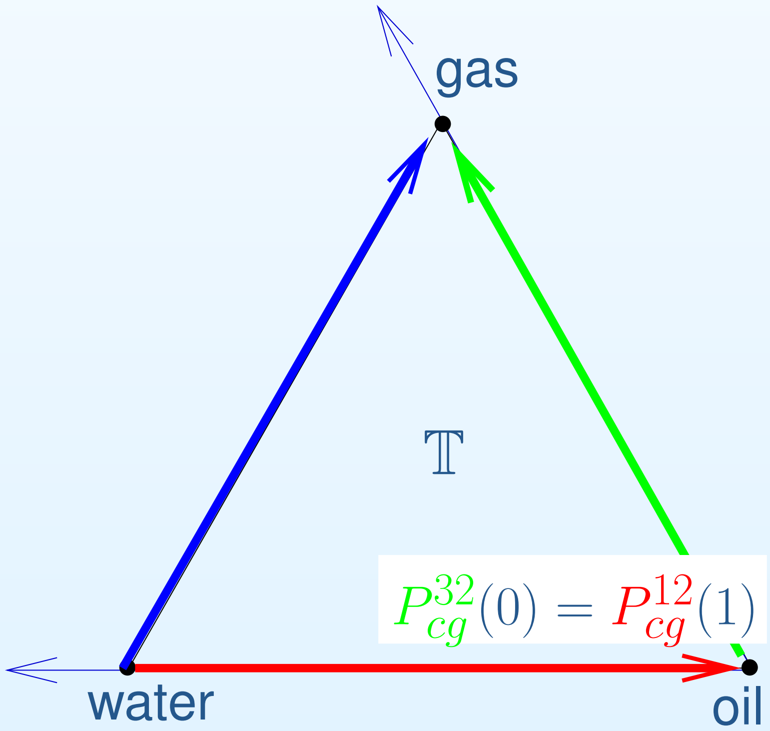
- on the water-gas side $\Longrightarrow$ P_{cg}^{13}
- on the water-oil side $\Longrightarrow$ P_{cg}^{12}
- on the gas-oil side $\Longrightarrow$ P_{cg}^{23}



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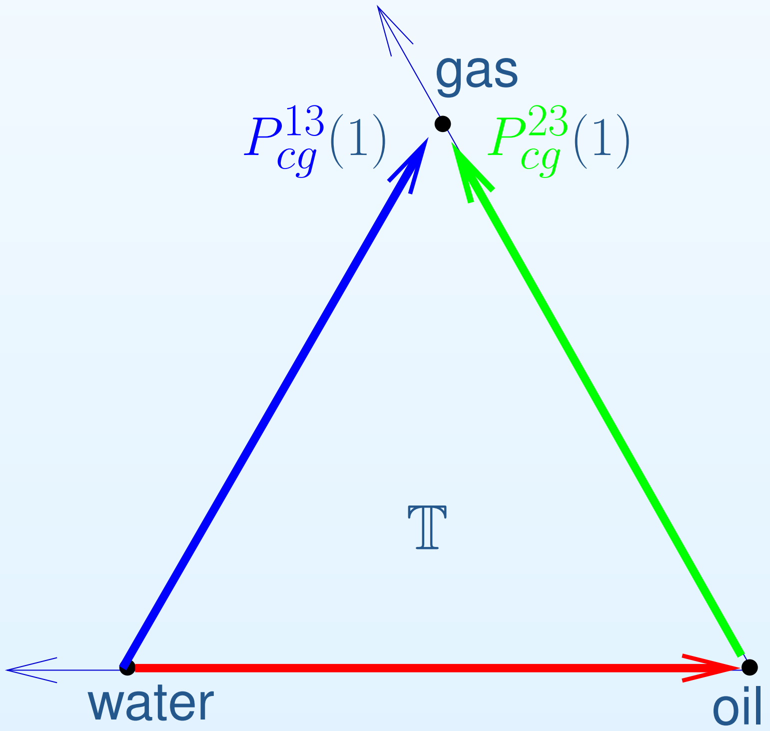
- Use (1) to determine of P_{cg} on $\partial\mathbb{T}$:

- on the water-gas side $\Longrightarrow P_{cg}^{13}$
- on the water-oil side $\Longrightarrow P_{cg}^{12}$
- on the gas-oil side $\Longrightarrow P_{cg}^{23}$



- TD-compatibility condition
  for the two-phase kr data :

$$P_{cg}^{13}(1) = P_{cg}^{23}(1) \ (= +\infty \ !)$$

Honoring two-phase data : 1 Determine P_{cg} on $\partial\mathbb{T}$

- Adjust P_{cg}^{32} to satisfy the TD-compatibility condition $P_{cg}^{13}(1) = P_{cg}^{32}(1)$

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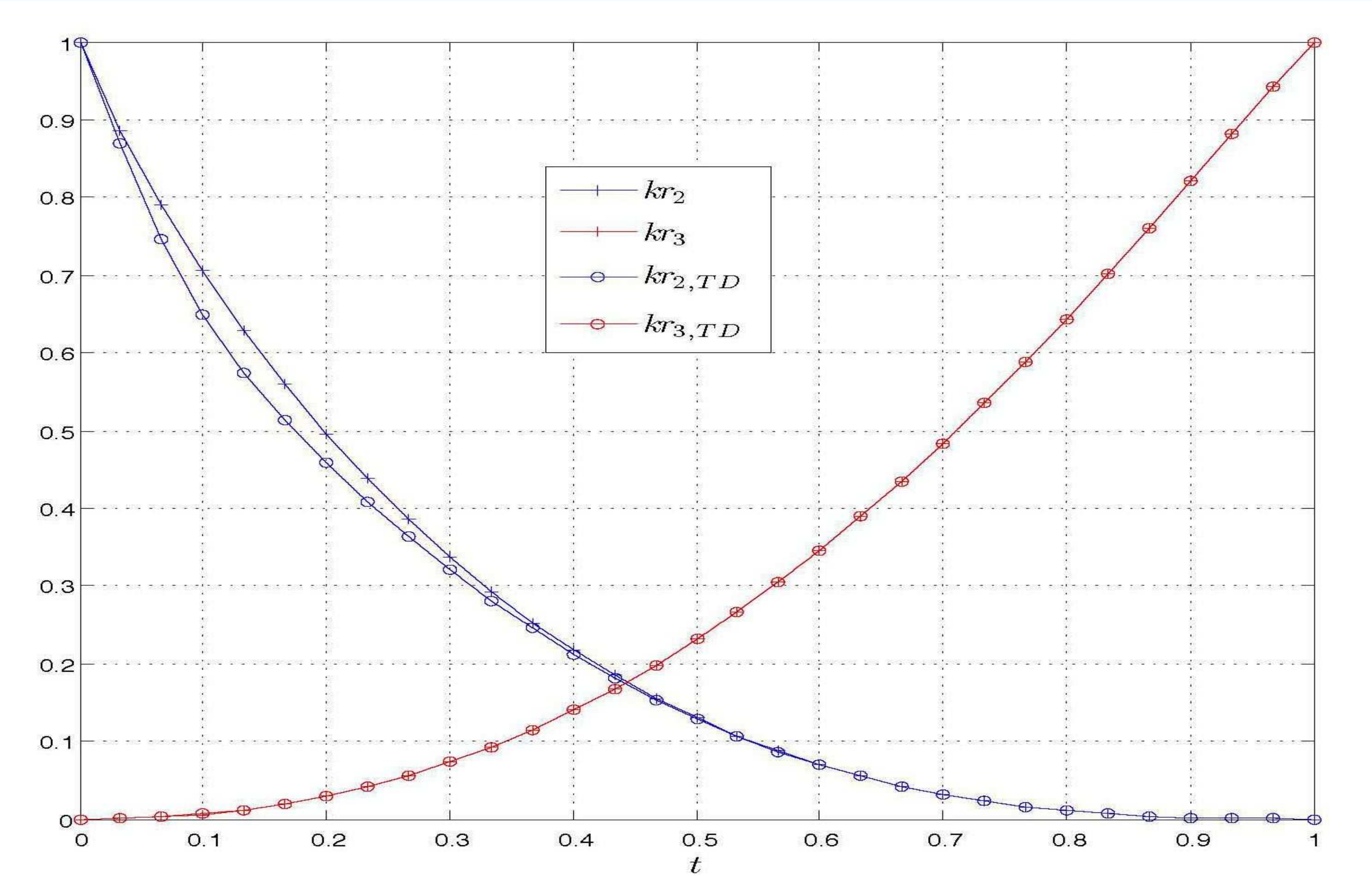
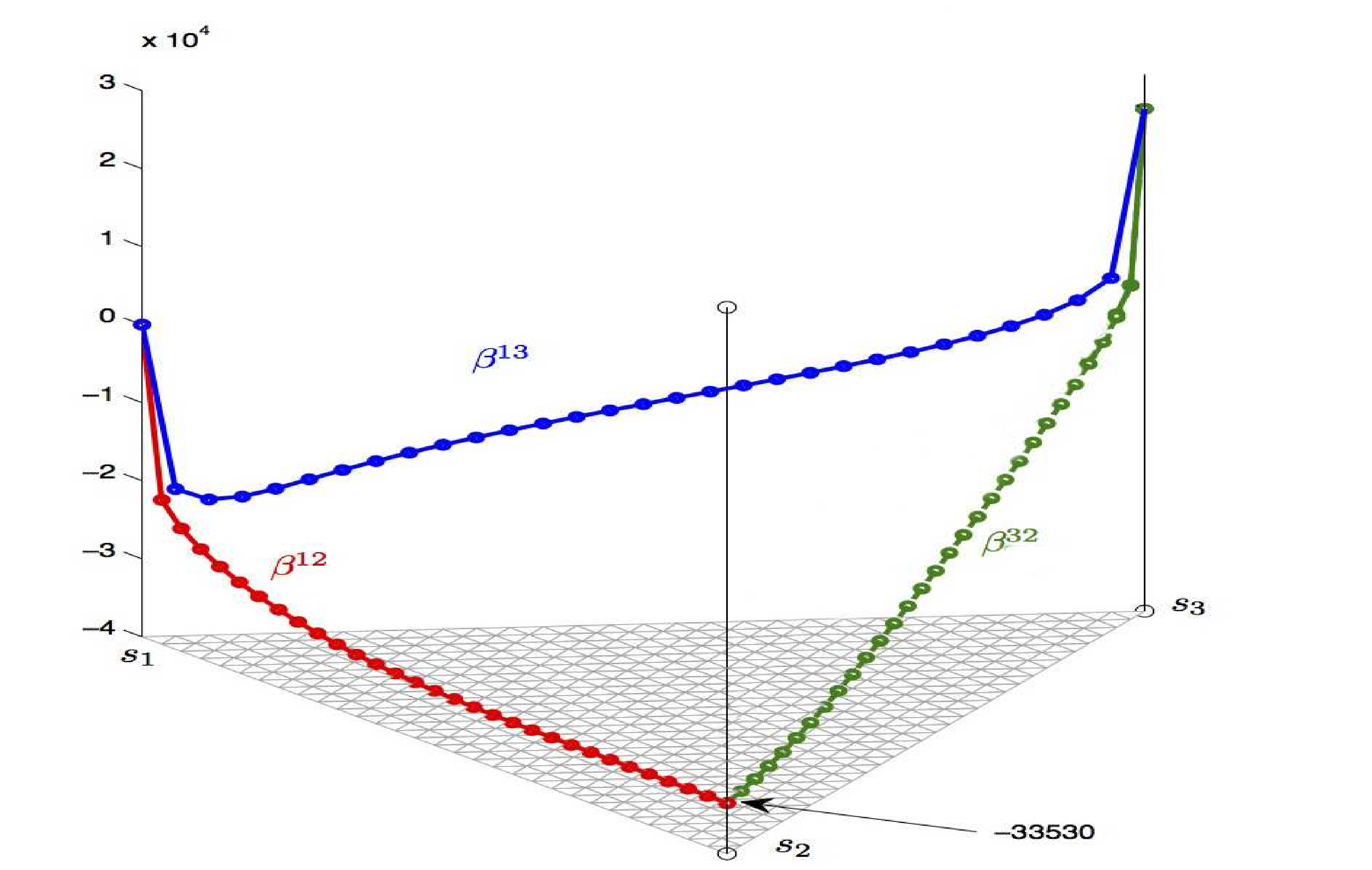


Honoring two-phase data : 1 Determine P_{cg} on $\partial\mathbb{T}$

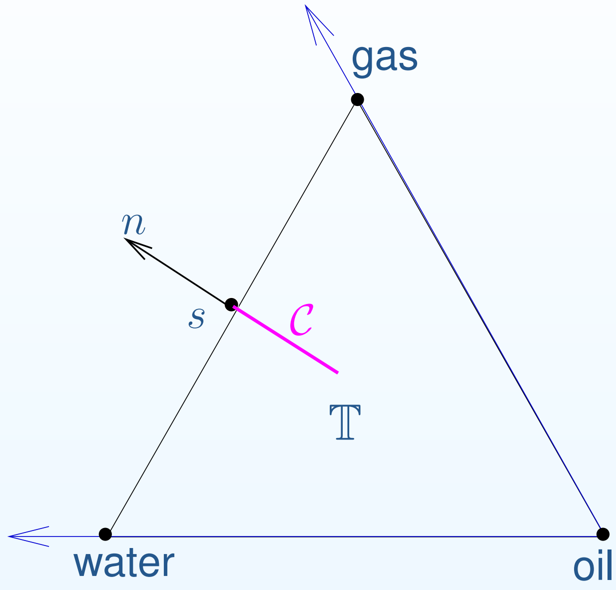
- now P_{cg} is available on $\partial\mathbb{T}$!



Honoring two-phase data: 2 determine $\partial P_{cg}/\partial n$ on $\partial\mathbb{T}$

- Let p be a given global pressure level.
- Let $s \in \partial\mathbb{T}$, n normal to $\partial\mathbb{T}$ at s, and define :

  $\mathcal{C} : t \in [0, \epsilon] \rightsquigarrow \mathcal{C}(t) = s - tn \in \mathbb{T}$
  a curve normal to $\partial\mathbb{T}$ at s.



Then $$\frac{\partial P_{cg}}{\partial n}(s) = -\frac{d}{dt}P_{cg}(\mathcal{C}(t))|_{t=0}.$$

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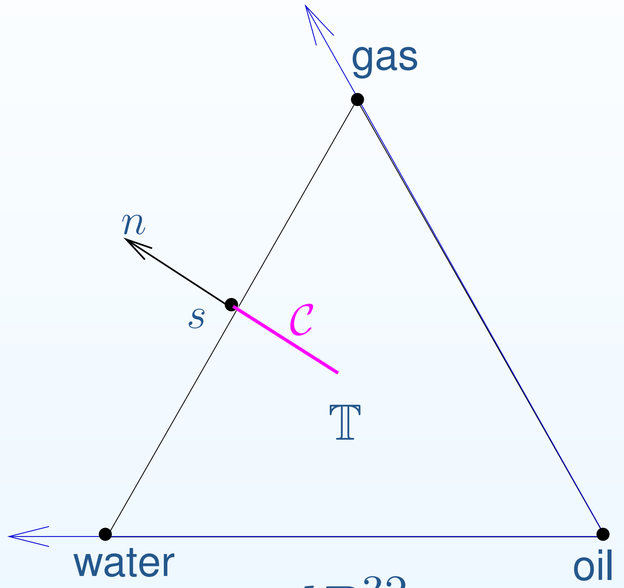

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But using again equation (1) :

$$(1)\ \frac{dP_{cg}}{dt} = f_1(\mathcal{C}, \underbrace{p - P_{cg}}_{P_2}) \frac{dP_c^{12}}{ds_1}(\mathcal{C}_1)\, \mathcal{C}_1' + f_3(\mathcal{C}, \underbrace{p - P_{cg}}_{P_2}) \frac{dP_c^{32}}{ds_3}(\mathcal{C}_3)\, \mathcal{C}_3' \ ,$$

gives:

$$\frac{\partial P_{cg}}{\partial n} = \begin{cases} \frac{\sqrt{3}}{3}\, f_1^{12}\, \frac{dP_c^{12}}{ds_1} & \text{(water-oil edge)}, \\ \frac{\sqrt{3}}{3}\left(f_1^{13}\frac{dP_c^{12}}{ds_1} + f_3^{13}\frac{dP_c^{32}}{ds_3}\right) & \text{(water-gas edge)}, \\ \frac{\sqrt{3}}{3}\, f_3^{23}\, \frac{dP_c^{32}}{ds_3} & \text{(gas oil edge)}, \end{cases}$$

Honoring two-phase data: 3 determine mobility d on $\partial\mathbb{T}$

$$d = \begin{cases} kr_1\, d_1(p - P_{cg}^{12} + P_c^{12}) + kr_2\, d_2(p - P_{cg}^{12}) & \text{(water-oil)} \\ kr_1\, d_1(p - P_{cg}^{13} + P_c^{12}) + kr_3\, d_3(p - P_{cg}^{13} + P_c^{32}) & \text{(gas-water)} \\ kr_3\, d_3(p - P_{cg}^{32} + P_c^{32}) + kr_2^{32}\, d_2(p - P_{cg}^{32}) & \text{(gas-oil)} \end{cases}$$

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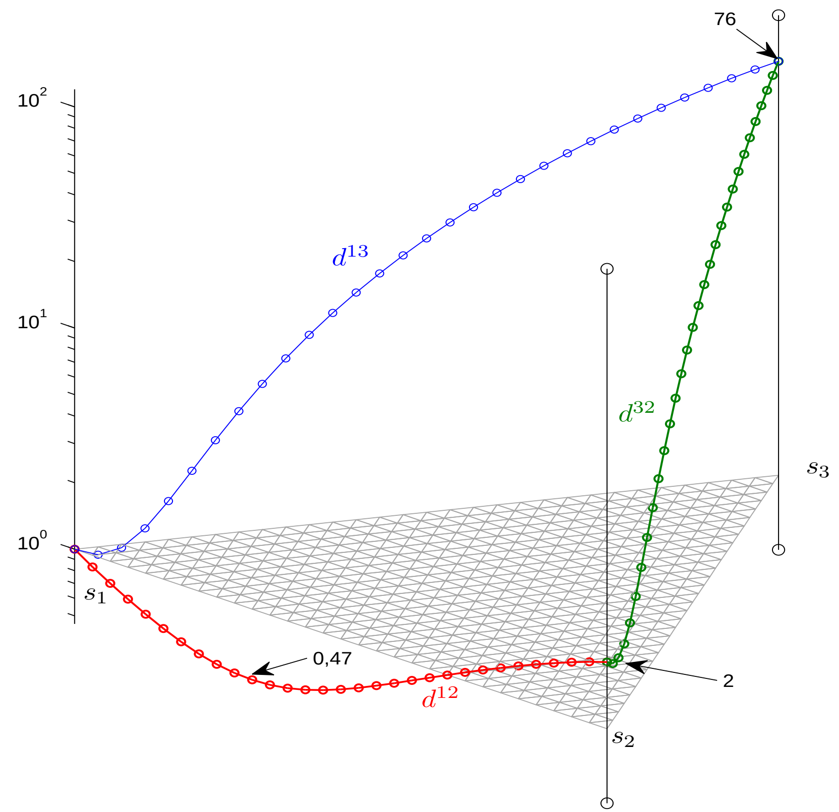

Honoring two-phase data: 4 - Find P_{cg} and d inside $\mathbb{T}$

A - FIRST APPROACH : INTERPOLATION

- Harmonic for d and Biharmonic for P_{cg} :

$$\left\{\begin{array}{llll} -\Delta d & = & 0 & \text{in } \mathbb{T}\,, \\ d & = & d^{\text{data}} & \text{on } \partial\mathbb{T}\,. \end{array}\right. \qquad \left\{\begin{array}{llll} \Delta^2 P_{cg} & = & 0 & \text{in } \mathbb{T}\,, \\ P_{cg} & = & P_{cg}^{\text{data}} & \text{on } \partial\mathbb{T}\,, \\ \dfrac{\partial P_{cg}}{\partial n} & = & \dfrac{\partial P_{cg}}{\partial n}^{\text{data}} & \text{on } \partial\mathbb{T}\,. \end{array}\right.$$

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- Finite element parameterization :

reduced HCT for P_{cg} , P^1 for d.

(di Chiara Roupert, Chavent and Schaefer, J. Comp. Physic 2010)

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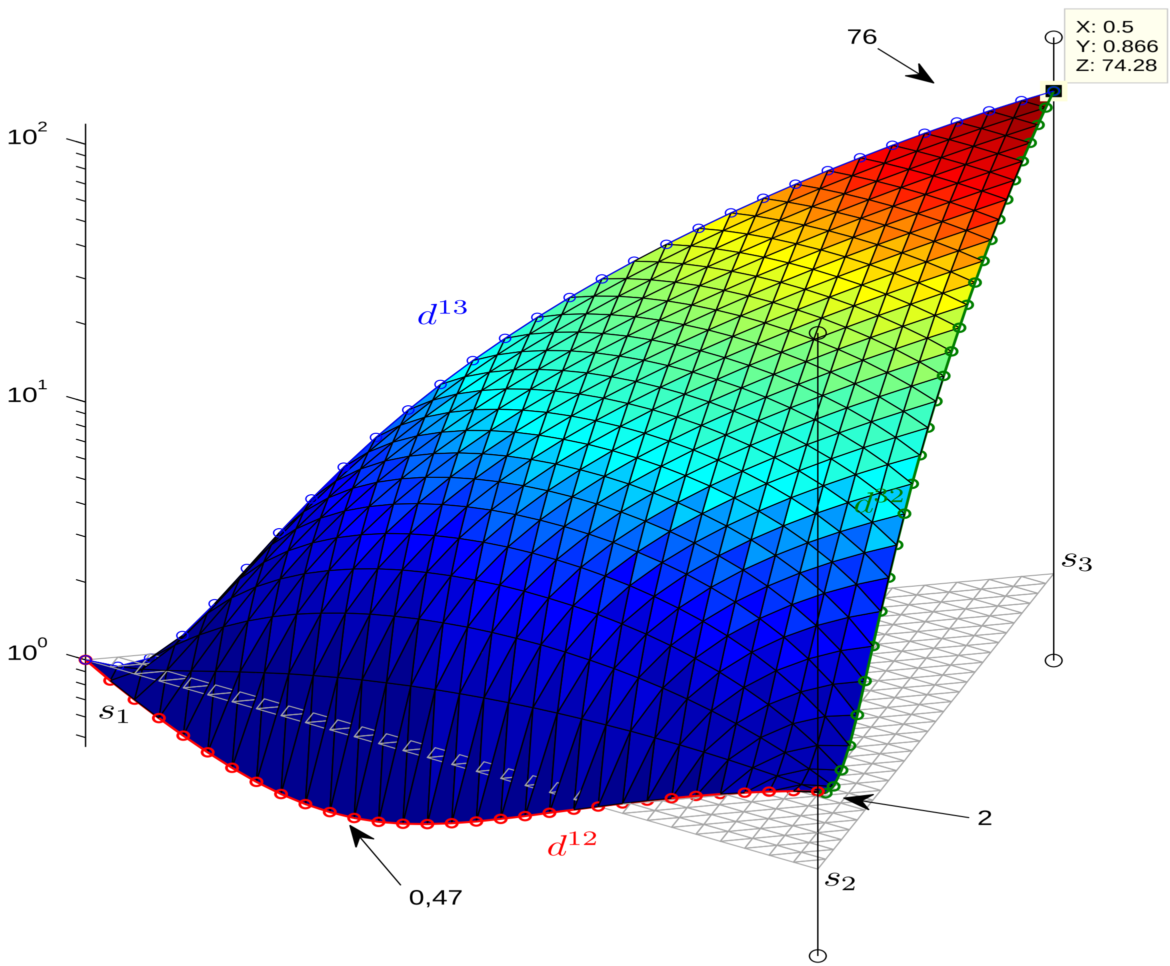
- Limitations : no control on relative permeabilities inside $\mathbb{T}$

B - SECOND APPROACH : OPTIMIZATION ...still to be tested !

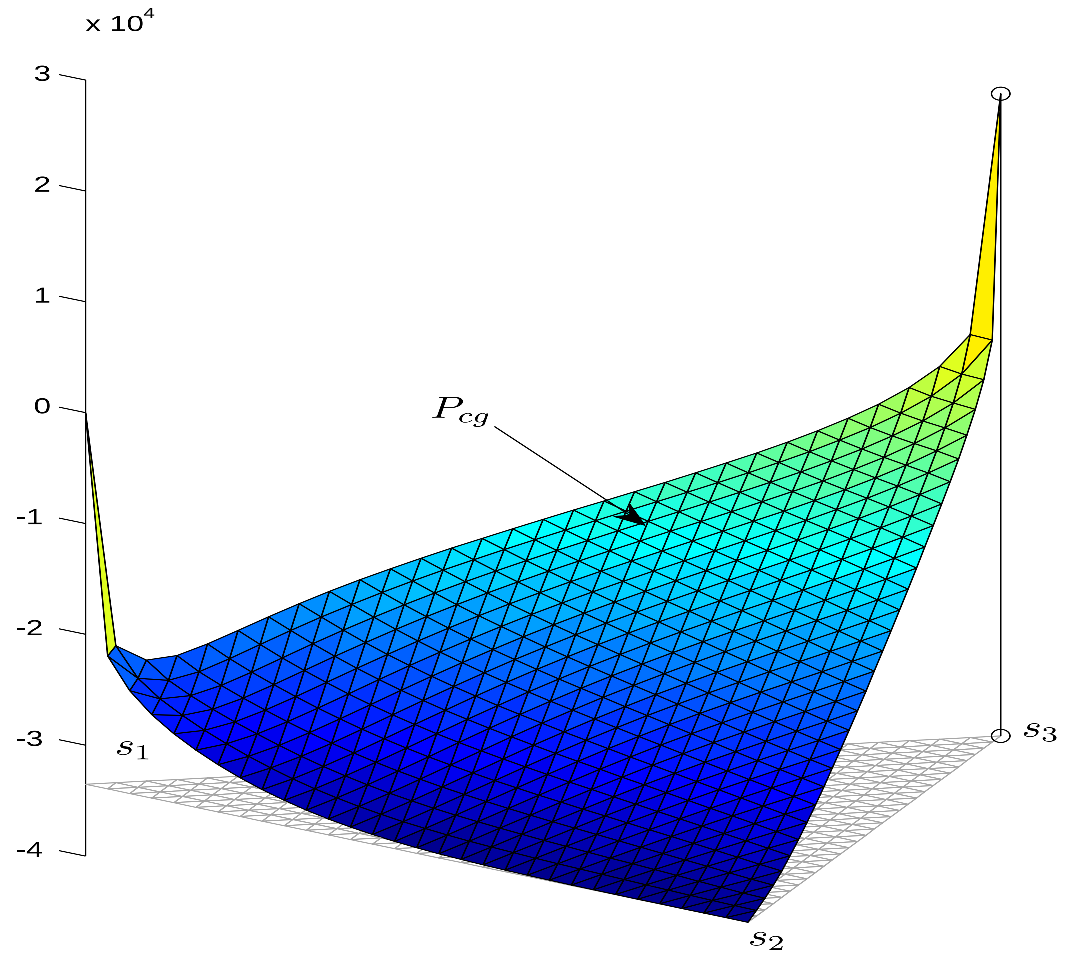
INTERPOLATION - Numerical results :

- global mobility d :



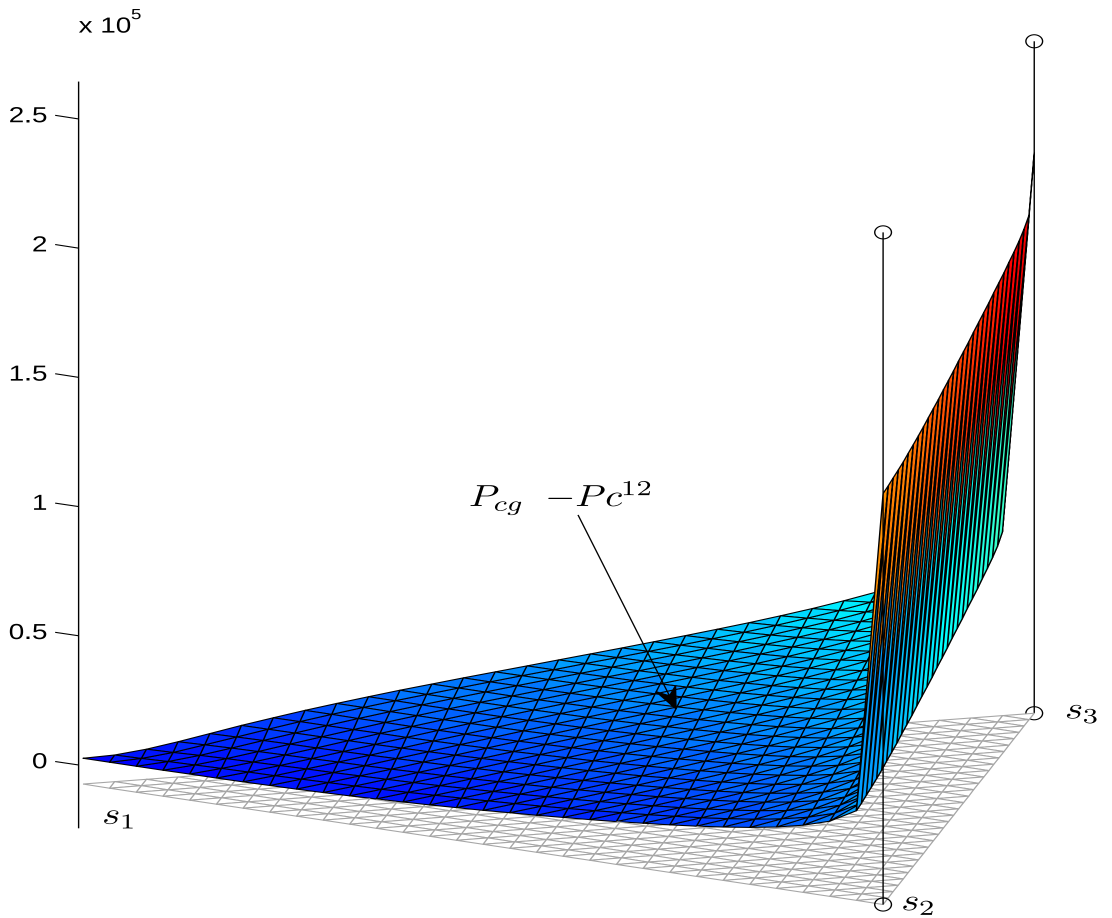
INTERPOLATION - Numerical results :

- global capillary pressure P_{cg} = global pressure p - oil pressure p_2



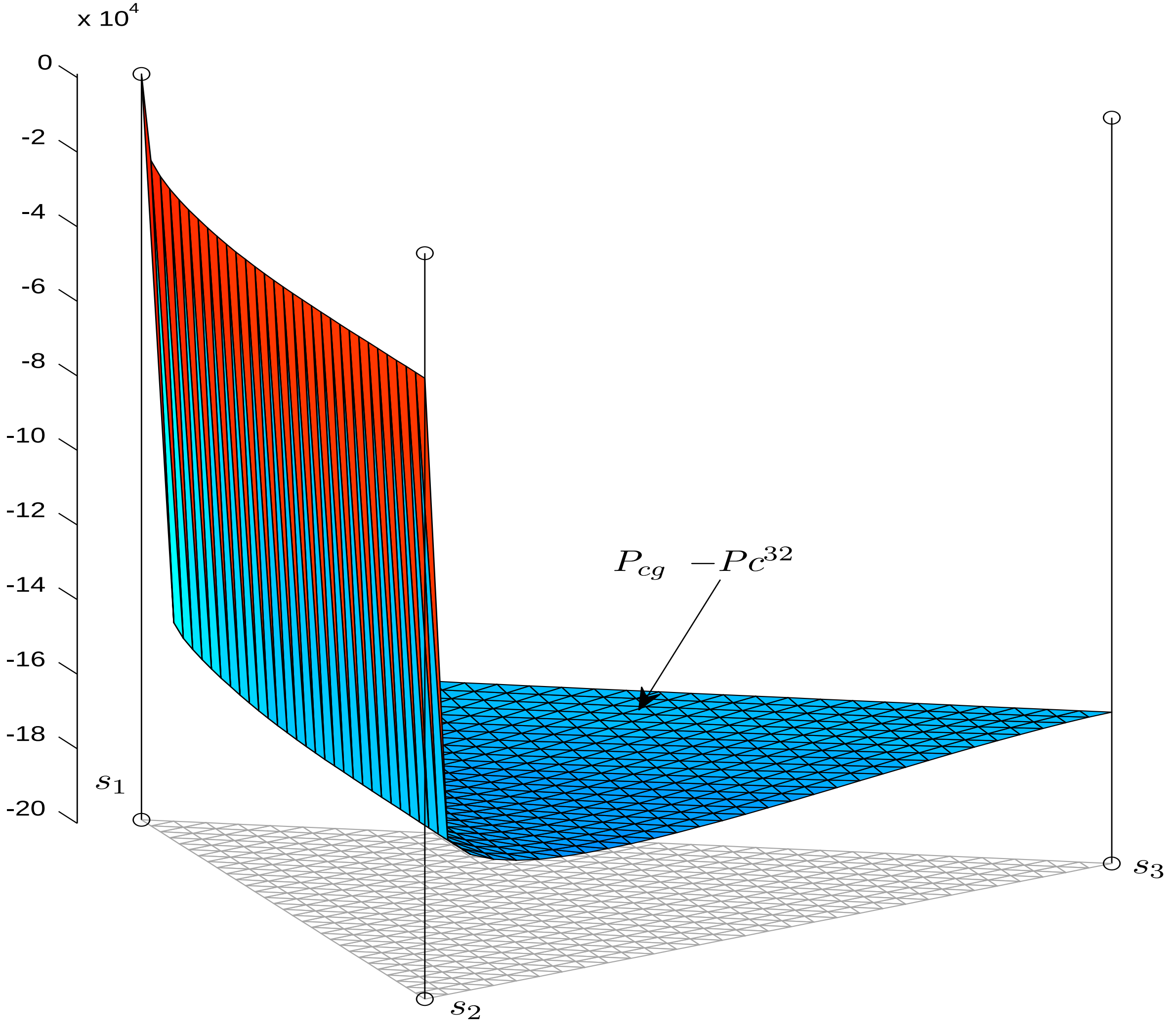
INTERPOLATION - Numerical results :

- global pressure p - water pressure $p_1 = P_{cg} - p_c^{12}$



INTERPOLATION - Numerical results :

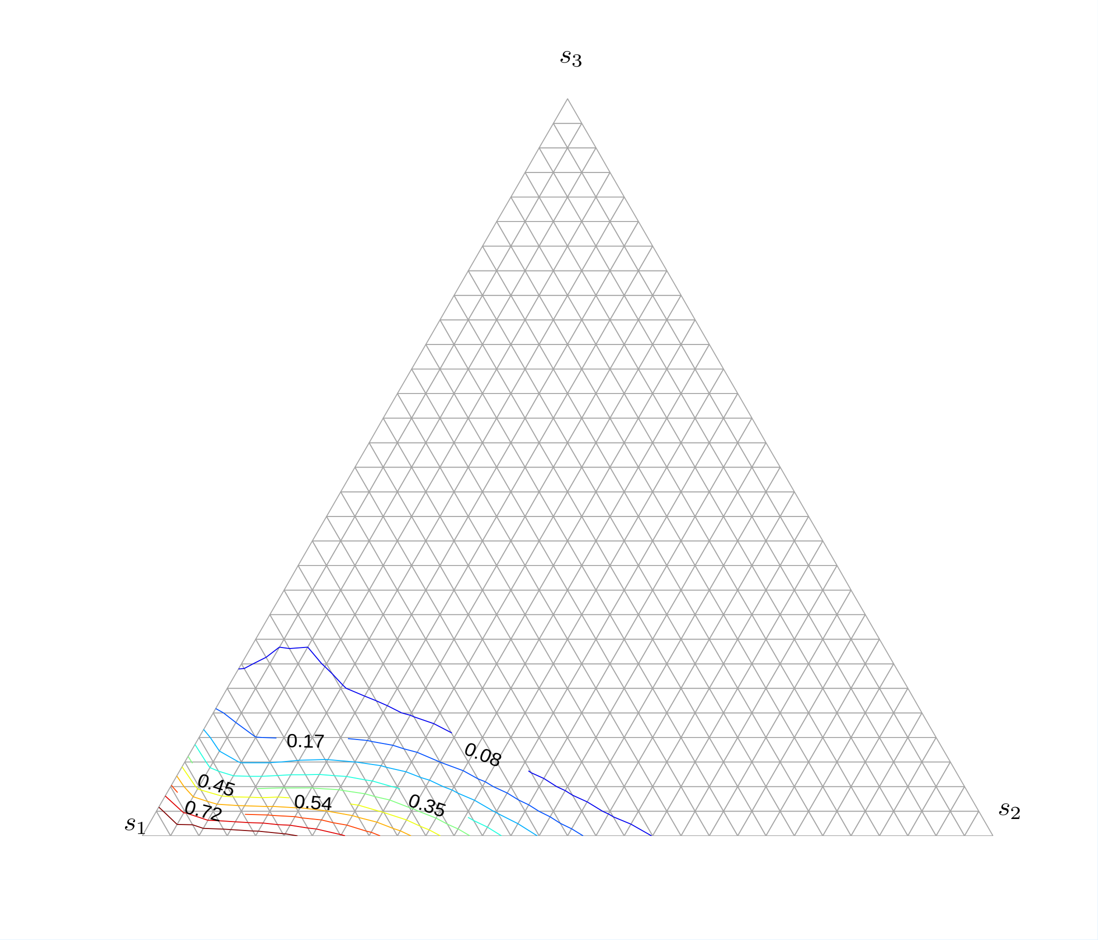
- global pressure p - gas pressure p_3 = $P_{cg} - p_c^{32}$



INTERPOLATION - Back to relative permeabilities :

- Step 1 : P_{cg} $\Rightarrow$ TD Three-phase fractional flows

$$\nu_j = \partial P_{cg} / \partial s_j \Big/ \, dP_c^{j2} / ds_j \; , \; j = 1, 3 \quad , \quad \nu_2 = 1 - \nu_1 - \nu_3$$



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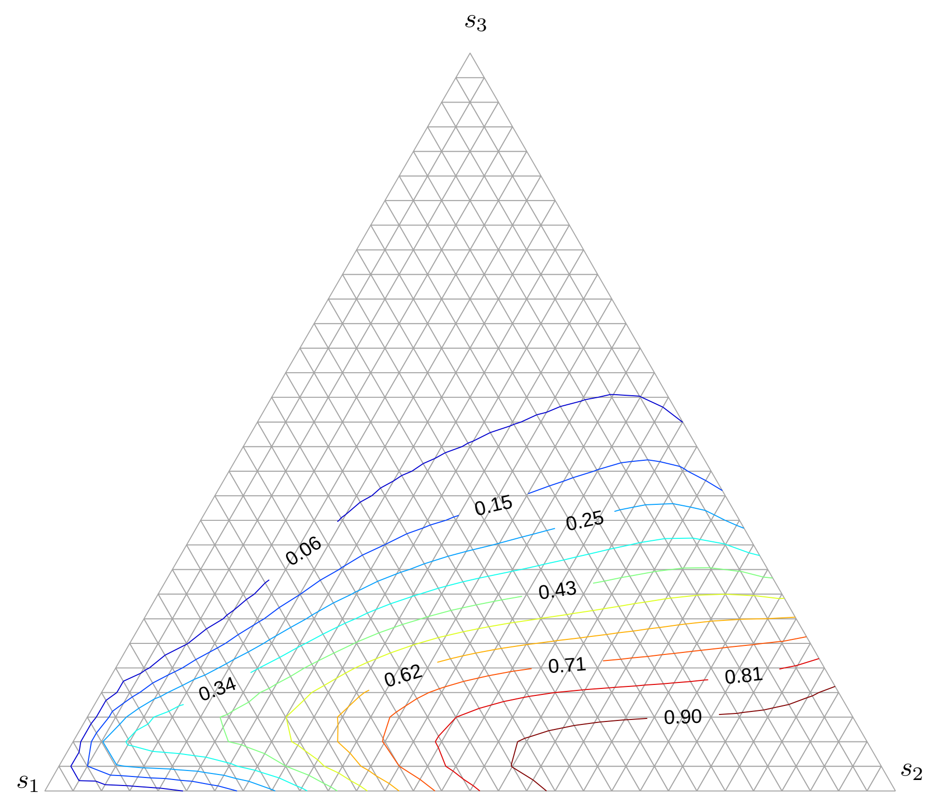

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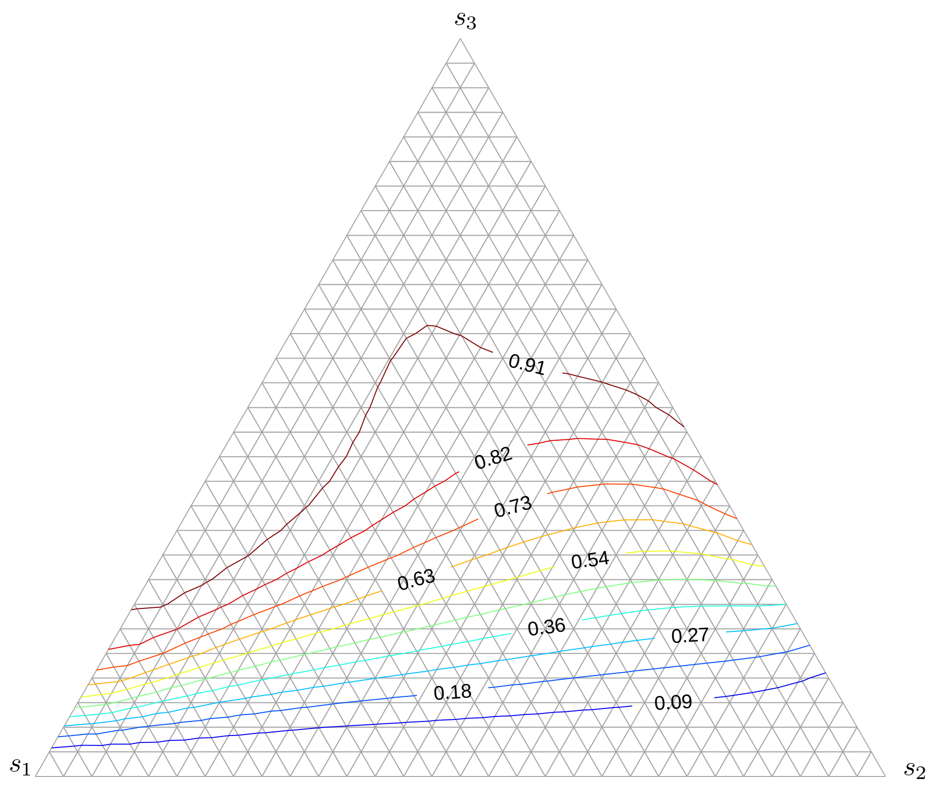
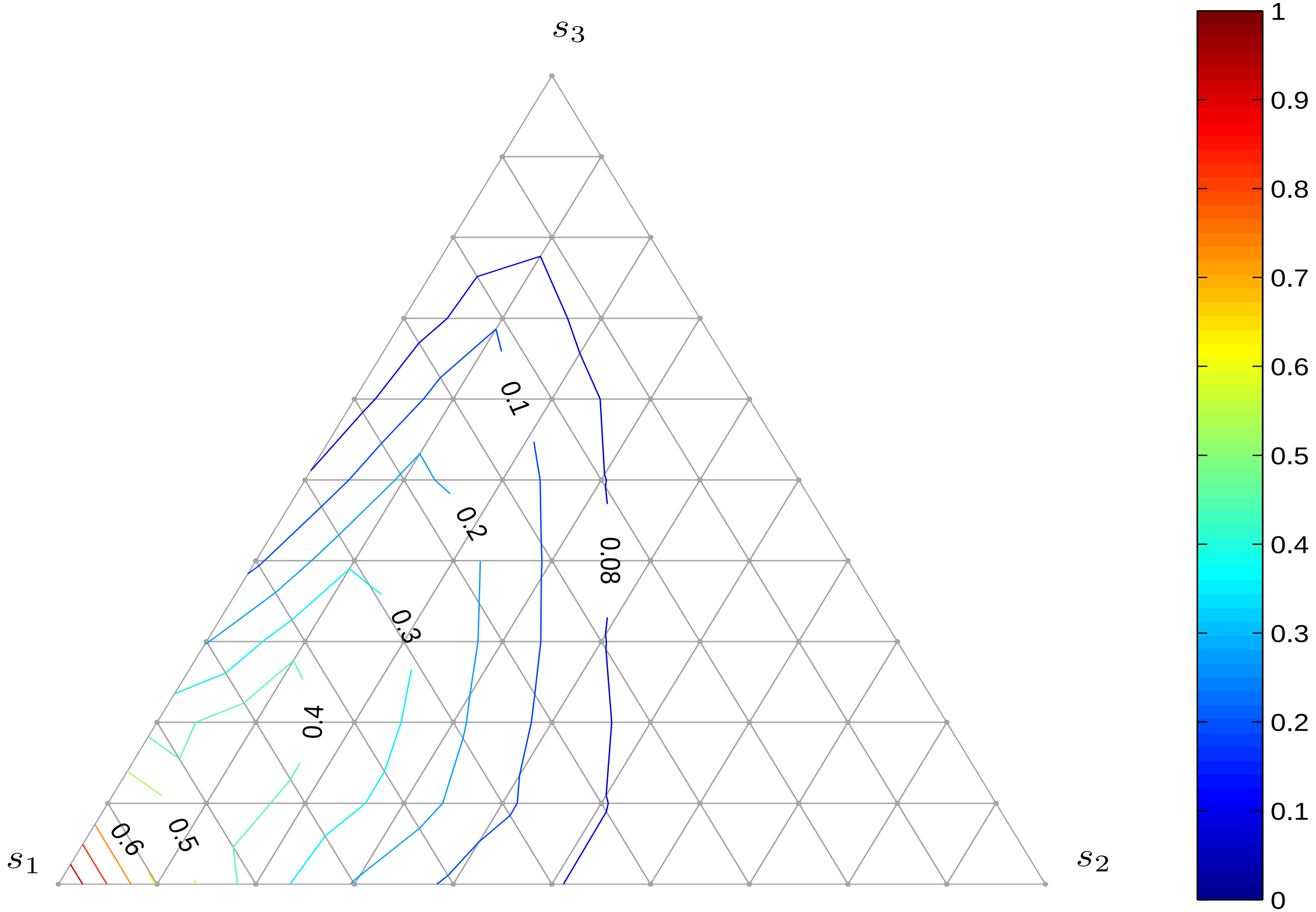


INTERPOLATION - Back to relative permeabilities :

- Step 2 : $P_{cg}, d \Rightarrow$ TD Three-phase relative permeabilities



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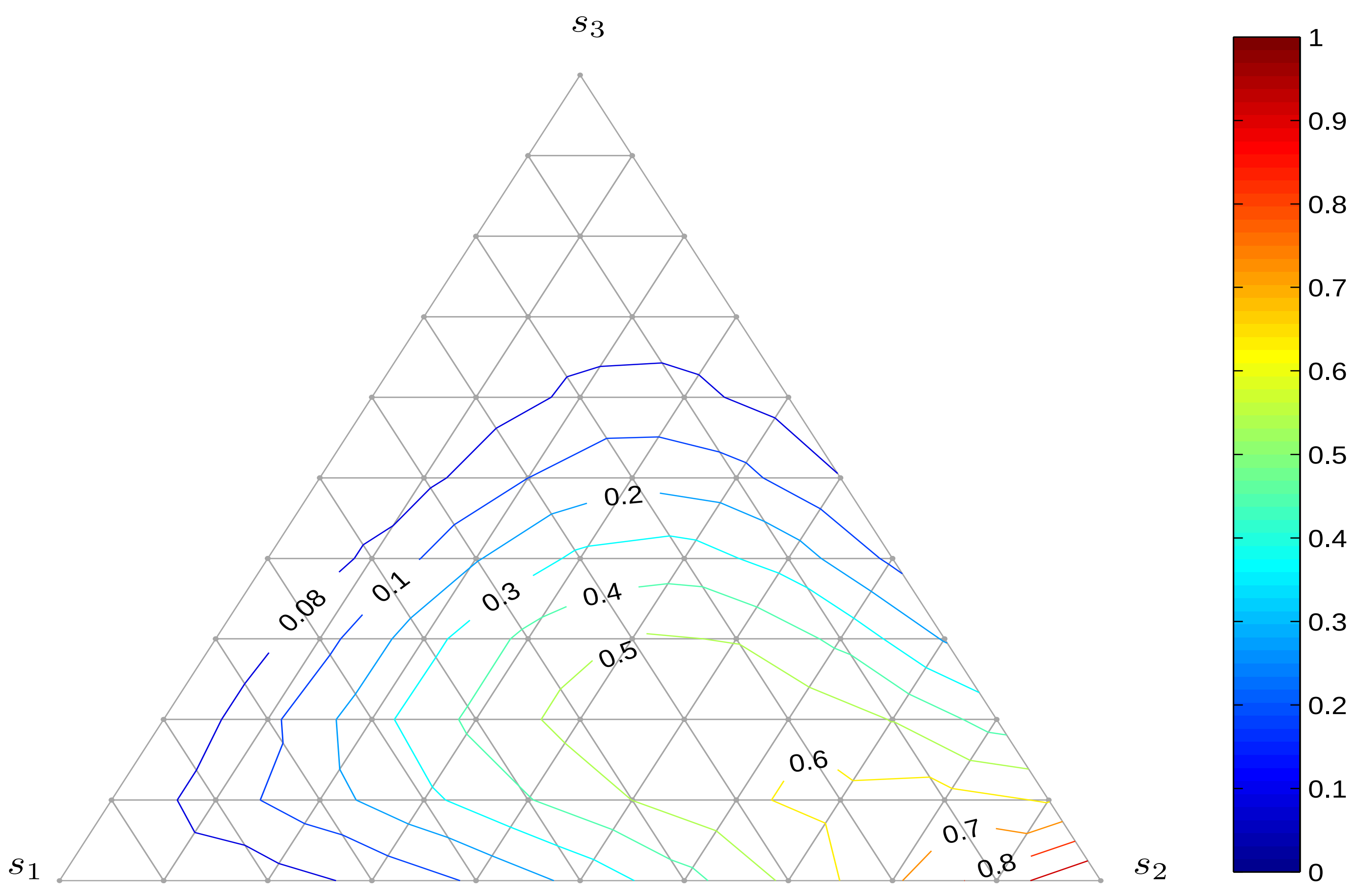

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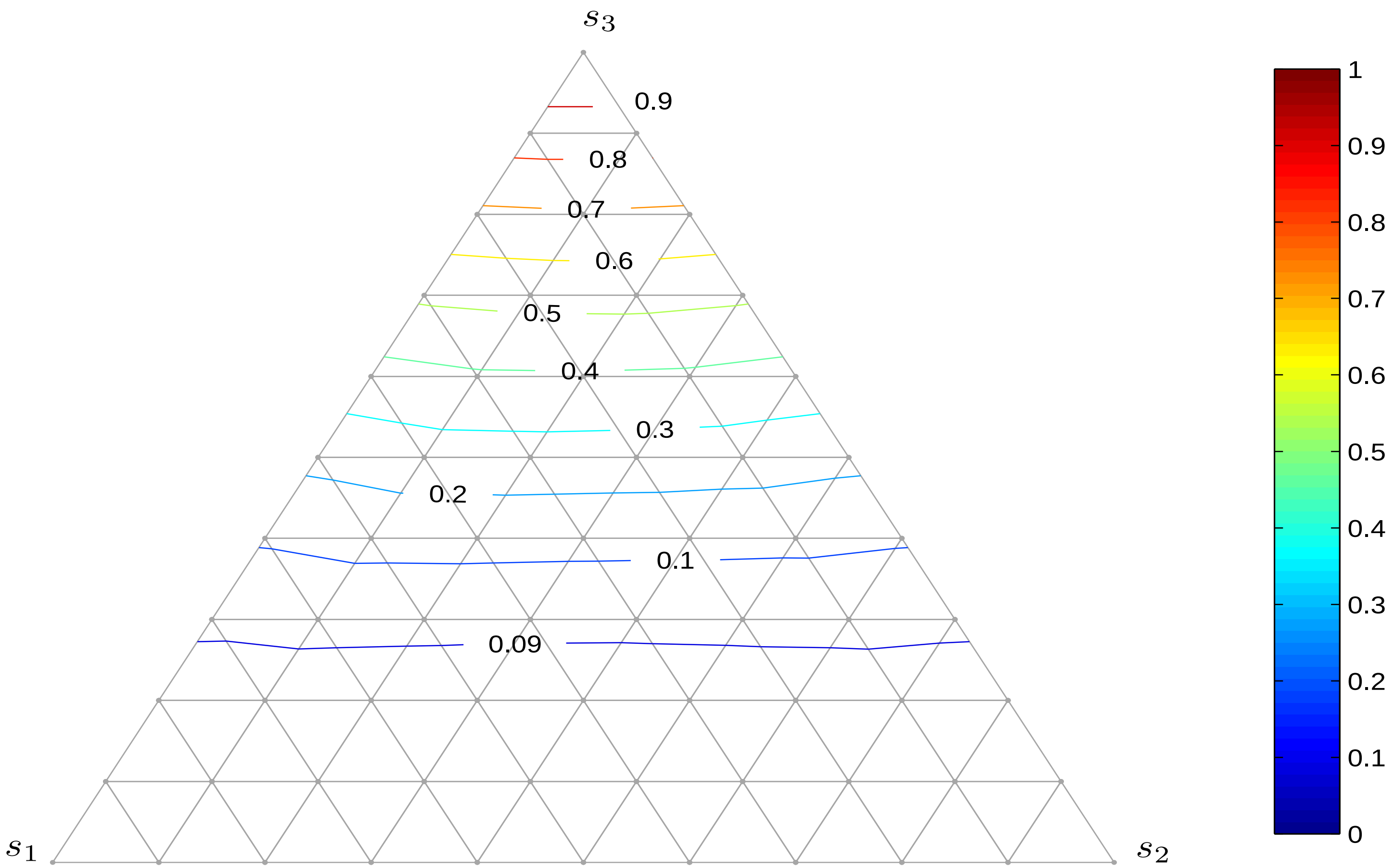

Conclusion

For three-phase compressible flows :

- Equivalent global pressure formulation :

  no gradient of capillary pressure in pressure equation !
- Well defined transformation ($0 \leq \partial P_{cg} / \partial p < 1$)
- the global pressure p is a good numerical unknown !

  remains smooth when the mobility of one fluid vanishes

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Bad news : three-phase data have to satisfy a TD condition
Good news : TD relative permeabilities can be INTERPOLATED
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Next step : replace INTERPOLATION by OPTIMIZATION :

- take constraints into account,
- try to match $kr_j^{\text{target}}(s), j = 1 \dots 3$.