Getting the Darcy law by time rescaling: a quick way to derive models for porous media from "first principles"

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Modeling and Simulation in Porous Media

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Deriving the Darcy law from the Euler equations

In 1755/57, Euler introduced the first "field theory" in Physics, and, at the same time, the first nonlinear PDE ever written

$$\partial_t \rho + \mathrm{div}(\rho \mathbf{v}) = \mathbf{0}, \quad \partial_t(\rho \mathbf{v}) + \mathrm{div}(\rho \mathbf{v} \otimes \mathbf{v}) = -\mathrm{grad}\mathbf{p}$$

where $(\rho, \mathbf{p}, \mathbf{v}) \in \mathbb{R}^{1+1+3}$ are the density, pressure and velocity fields of a fluid and p is assumed to be a given function of ρ.


PRO JUVENTUTE 1957
$e^{i\varphi} = \cos\varphi + i \sin\varphi$
LEONHARD EULER 1707-1783
5
+5 HELVETIA

XXI. Nous n'avons donc qu'à égaler ces forces accélératrices avec les accélerations actuelles quē nous venons de trouver, & nous obtiendrons les trois équations ſuivantes :

$$P - \frac{1}{q}\left(\frac{dp}{dx}\right) = \left(\frac{du}{dt}\right) + u\left(\frac{du}{dx}\right) + v\left(\frac{du}{dy}\right) + w\left(\frac{du}{dz}\right)$$

$$Q - \frac{1}{q}\left(\frac{dp}{dy}\right) = \left(\frac{dv}{dt}\right) + u\left(\frac{dv}{dx}\right) + v\left(\frac{dv}{dy}\right) + w\left(\frac{dv}{dz}\right)$$

$$R - \frac{1}{q}\left(\frac{dp}{dz}\right) = \left(\frac{dw}{dt}\right) + u\left(\frac{dw}{dx}\right) + v\left(\frac{dw}{dy}\right) + w\left(\frac{dw}{dz}\right)$$

Si nous ajoutons à ces trois équations premièrement celle, que nous a fournie la conſidération de la continuité du fluide :

$$\left(\frac{dq}{dt}\right)+\left(\frac{d.qu}{dx}\right)+\left(\frac{d.qv}{dy}\right)+\left(\frac{d.qw}{dz}\right)=0.$$

Si le fluide n'étoit pas compreſſible, la denſité q ſeroit la même en Z, & en Z′, & pour ce cas on auroit cette équation :

$$\left(\frac{du}{dx}\right)+\left(\frac{dv}{dy}\right)+\left(\frac{dw}{dz}\right)=0.$$

qui eſt auſſi celle ſur laquelle j'ai établi mon Mémoire latin allégué ci-deſſus.

An unusual but simple trick to get the Darcy law out of the Euler equations consists in rescaling time in a nonlinear way as follows

$$t \to \tau = t^2/2, \quad (\rho, v)(t, x) \to (\rho(\tau, x), \tau' v(\tau, x)), \quad \tau' = \frac{d\tau}{dt} = t$$

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By dropping the terms written in red* we indeed get the Darcy law

$$\boxed{\rho v = -\mathrm{grad} p, \quad \partial_\tau\rho = \triangle p}$$

(*) motivation: $(\tau')^{\mathbf{2}} = \mathbf{2}\tau = \mathbf{t}^{\mathbf{2}} << \mathbf{1} = \tau''$ **for short times.**

A richer example: "Darcy's Magnetohydrodynamics"

$$\partial_t \rho + \mathrm{div}(\rho \mathbf{v}) = \mathbf{0}, \quad \rho \mathbf{v} = \mathrm{div}(\eta \mathbf{B} \otimes \mathbf{B}) - \mathrm{grad}\mathbf{p}$$

$$\partial_t \mathbf{B} + \mathrm{curl}(\mathbf{B} \times \mathbf{v}) + \mathrm{curl}(\mu \ \mathrm{curl}(\nu \mathbf{B})) = \mathbf{0}$$

Here $(\rho, \mathbf{p}, \mathbf{v}, \mathbf{B}) \in \mathbb{R}^{1+1+3+3}$ are the density, pressure, velocity and magnetic fields, $(\mu, \nu, \eta, \mathbf{p})$ being given functions of ρ.

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A priori, such a model, combining Darcy law and MHD, is very far from "first principles" and must require many steps to be derived!

A direct derivation from..."pure" Geometry and Physics

Assume $\boxed{\mu = \nu = \eta = \rho^{-\mathbf{1}}}$ **with pressure law** $\boxed{\mathbf{p} = -\rho^{-\mathbf{1}}}$

("Chaplygin" pressure, used in Cosmology, with sound speed $(\frac{dp}{d\rho})^{1/2} = \rho^{-1}$**)**

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This can be derived, through the simple "Darcy time-rescaling" $t \to t^2/2$, from a very "pure" equation from Geometry and Physics, designed by Born and Infeld in 1934 and used in... String Theory!

Max BORN (1882-1970) Physics Nobel prize 1954

Geometric definition of the Born-Infeld theory

In a $d+1$ dimensional Lorentzian space-time manifold of metric $\boxed{g_{ij}dx^i dx^j}$ the BI theory involves closed 2-forms $\boxed{\mathcal{B} = \mathcal{B}_{ij}dx^i \wedge dx^j}$

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We will concentrate on the 3+1 Minkowski space of special relativity (as Max Born and Leopold Infeld did in 1934).

Born-Infeld in traditional electromagnetic notations

After simple but tedious calculations, the Born-Infeld equations read, using classical electromagnetic notations,

$$\partial_t \mathbf{B} + \mathrm{curl}\left(\frac{\mathbf{B} \times (\mathbf{D} \times \mathbf{B}) + \mathbf{D}}{\sqrt{1 + \mathbf{D}^2 + \mathbf{B}^2 + (\mathbf{D} \times \mathbf{B})^2}}\right) = \mathbf{0}, \quad \mathrm{div}\mathbf{B} = \mathbf{0}$$

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They return the usual homogeneous Maxwell equations in the vacuum (terms in black) as B, D are fields or very small amplitude.

The "energy-momentum" conservation laws

By Noether's theorem*, we get 4 extra conservation laws

$$\partial_t \rho + \mathrm{div}(\rho \mathbf{v}) = \mathbf{0}, \quad \partial_t(\rho \mathbf{v}) + \mathrm{div}(\rho \mathbf{v} \otimes \mathbf{v} - \frac{\mathbf{B} \otimes \mathbf{B} - \mathbf{D} \otimes \mathbf{D}}{\rho}) = \mathrm{grad}(\rho^{-1})$$

$$\mathbf{v} = \frac{\mathbf{D} \times \mathbf{B}}{\rho}, \quad \rho = \sqrt{1 + \mathbf{D}^2 + \mathbf{B}^2 + (\mathbf{D} \times \mathbf{B})^2}$$

*** which can be applied here since, once the Minkowski space-time is chosen , the Born-Infeld action gets invariant under time and space translations/**

The augmented 10x10 Born-Infeld system

(following Y.B. ARMA 2004 and Y.B./Weinan Yong JMP 2006)

It is consistent (and much simpler) to consider $(\mathbf{B}, \mathbf{D}, \rho, \mathbf{v})$ as *independent* variables solving the 10×10 augmented system (which includes the 4 extra conservation laws)

$$\partial_t \mathbf{B} + \mathrm{curl}(\mathbf{B} \times \mathbf{v} + \rho^{-1}\mathbf{D}) = 0, \quad \partial_t \mathbf{D} + \mathrm{curl}(\mathbf{D} \times \mathbf{v} - \rho^{-1}\mathbf{B}) = 0$$

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while ignoring the algebraic constraint $v = \frac{D \times B}{\rho}, \quad \rho = (1 + D^2 + B^2 + (D \times B)^2)^{1/2}$.

Darcy time-rescaling of the (augmented) BI equations

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Ignoring the terms written in red leads to the desired model of Darcy MHD, with "constitutive laws" $\mu = \nu = \eta = -p = 1/\rho$ **!**

Energy conservation versus energy dissipation

In the (augmented) BI equations, the total energy is ***conserved***

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After dropping the red terms, we get the energy *dissipation* relation

$$\frac{d}{d\tau}\int \frac{1+B^2}{\rho}dx = -2\int \frac{D^2+(\rho v)^2}{\rho}dx$$

(remember: $(\tau')^2 = 2\tau = t^2 << 1 = \tau''$**)**

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